

GLOBAL JOURNAL OF ADVANCED ENGINEERING TECHNOLOGIES AND SCIENCES**SINE COSINE OPTIMIZATION-BASED APPROACH FOR OPTIMIZING THERMOELECTRIC MODULES IN WASTE HEAT RECOVERY SYSTEMS****Yashwardhan Singh Chandrawat, Khemraj Beragi**singhchandrawat84@gmail.com , khemrajberagi@gmail.comDOI: <https://doi.org/10.29121/gjaets.2025.9.4>**ABSTRACT**

In this paper, the topic of thermoelectric modules (TEMs) optimization in waste heat recovery systems is discussed, and it is addressed on the optimization of power to the maximum and efficiency to the optimum. The new strategy of employing the Sine Cosine Optimization (SCO) algorithm is proposed to optimize the most important parameters of TEMs, including length and cross-sectional area of thermoelectric legs. The objective of the study is to overcome the drawback of the conventional optimization methods, which include the Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), as they are time-consuming and inconvenient with respect to making real-time changes. Using dynamic adjustment of the dimension of thermoelectric legs according to certain temperature gradients, SCO algorithm will help to improve the functionality of TEMs in reality. The simulation results indicate that the SCO-optimized TEM is superior to the base TEM, giving it power generation enhancing results, where the output becomes 33% more with optimal current amounts. The study is a suggestive solution to the enhancement of thermoelectric waste heat recovery systems effectiveness and fastidiousness.

KEYWORDS: Cross-Sectional Area, Leg length, Seebeck Effect, Sine Cosine Optimization (SCO), Temperature Gradients, Thermoelectric Generator (TEG), Thermoelectric Module (TEM), Waste Heat Recovery.

1. INTRODUCTION

The use of thermoelectric modules (TEMs) in converting waste heat to electrical energy is a significant component in the present complex energy recovery efforts in the industry that provides an environment friendly approach. Nevertheless, TEMs generally do not operate optimally, which results in lower performance triggered by temperature gradients and thermoelectric leg dimensions. With industries wanting to reduce the amount of energy they waste and become more sustainable, the optimization of the performance of TEMs has already gained the prominence among the list of research topics. In the current paper, the idea of increasing the power generation capacity of such TEMs by use of Sine Cosine Optimization (SCO) algorithm is presented with reference to waste heat recovery systems. Voltage production and internal resistance are two issues that SCO targets to balance by dynamically optimizing the leg dimensions, length and cross-sectional area, through the SCO algorithm in order to achieve maximum efficiency and power output. Thermoelectric module optimization is important when it comes to improving its performance in real life situations where temperature ranges, and weather, varies throughout. Current methods of optimization, including Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), fail to comply with real time corrections and computationally extensive requirements. This study resolves the shortcomings of these studies through the use of dynamic and real-time optimization model incorporating SCO. The model is adjustable to change in operational environment and hence maintaining a state of the thermoelectric system to be efficient and responsive. With multiple simulations carried out, the present research shows the versatility of the SCO algorithm when it comes to realization of a TEM with enhanced power output and efficiency as a whole system.

2. LITERATURE REVIEW

In [1], the researchers developed a new more optimal method of improving thermoelectric modules (TEMs) that are used in the recovery of waste heat in fluids. The study observed the temperature non-uniformities of the thermoelectric legs as the said weaknesses reduce the module output current. The optimization method demands that the length of every thermoelectric leg should be assigned on the basis of its personal temperature difference. A fluid-thermal-electric multi-physics model was created by scientists to simulate various states of operation of the system and consider them. The new method produced high output power than standard thermoelectric modules. The study based on this model entailed the simplification of thermoelectric module layouts in the number of thermoelectric legs used in their analysis but these modeled designs may not match the real-life application and the computational overhead of the said optimization techniques may become very large with an increasing number of thermoelectric legs in large systems.

In [2], the authors evaluated the impact of temperature gradient in thermoelectric materials through wastes heat recovery systems. The authors examined the methods of material optimization and the role of material properties in improving output of thermoelectric generator. Their research is very important in the material selection in order to improve the performance of the system. This paper missed the opportunity to present important implementation limitations that manifest themselves in industrial scale up of these materials when going to scale systems to be used practically even though these changes in heat flux are important or the material breakdown occurring with time.

In [3] the authors studied the possibilities of artificial neural network (ANN) to boost the waste heat recovery efficiency of thermoelectric generators. The ANN was fed with experimental data that subsequently could be used to predict various conditions under which the system was supposed to operate. The model worked well in estimating behavior of systems performance. The only disadvantage of this method is the large size of the training set used to develop ANN since novel and untested environmental changes would require permanent retraining of ANNs to achieve the optimal performance of ANN.

The researchers of [4] worked in order to enhance thermoelectric generator systems in order to administer thermogenerator thermal management systems. The scholars used a two-pronged CFD and optimization approach to design heat exchanger to ensure improved thermoelectric system performance. The simulation of such proposed solution held a promising result yet implementing such methods to real time systems experiencing multiple parameters of operations would be a challenge since the implementation process is complex.

In [5] an optimization model of thermoelectric module optimized by genetic algorithm computation and multi-objectives of optimization was developed. Their research results meant that the optimized thermoelectric module design had the chance to increase power output. The principal shortcoming of genetic algorithm is that they require a lot of computation and are inappropriate to optimize systems that require real time responses due to high rates of convergence.

The study in [6] assessed the performance of thermoelectric generator by design of a new method of numerical simulation. The model devised was a mixture of thermal and electrical properties and was advancing in the way of designing thermoelectric modules. The guaranteed results of the model were confined by the fact that it omitted the degrading effect of materials and all the environmental effects because these elements potentially affect the efficiency and durability of thermoelectric systems during real applications.

In the study done by [7], particle swarm optimization (PSO) was presented as the optimization utility. The researchers demonstrated that PSO was an efficient method in order to optimize the system parameters that also comprised of material properties as well as the leg dimensions in order to obtain maximum efficiency outcomes. The performance outcome of the PSO algorithm depends on the complexity of the system and optimization requirements hence resulting in possibly sub-optimal performance of highly non-linear systems.

Authors of [8] introduced nanostructured alloys which was a new thermoelectric material performing better in terms of efficiency parameters than conventional thermoelectric material based on data in [8]. The study of research proved the applicability of the current materials and to enhance the thermoelectric system. Costs of production and scalability challenges of the material are some of the greatest hindrances to its application in commercial thermoelectric generator throughout the market.

The authors of [9] provides evidence through which they demonstrate how thermoelectric systems relate with renewable energy sources when used in solar-thermal power systems. These authors demonstrated that it is necessary to combine the thermoelectric generators with the solar thermal systems in order to increase the overall system efficiency. The optimization of such an approach greatly relies on the weather conditions and intensity of sun radiations that leads to system performance that is unpredictable and difficult to control in the non-ideal times of its operation.

In a study that was carried out by the authors of [10], the authors used simulated annealing along with genetic algorithms in order to optimize the working of thermoelectric systems. In their studies, they revealed that the hybrid optimization algorithm proved capable of addressing the multifaceted optimization activity with multiple performance goals. Computational demand of the technique coupled with potential local minima causes it to struggle finding the global optimum solution of the much large systems.

The study carried out and reported in [11] evaluated thermoelectric-based systems used in the automotive waste-heat recovery. One company of researchers developed a model to come up with performance to ascertain that thermoelectric modules would be capable of prevailing in vehicle exhaust systems. The research team has not only made thermoelectric technology an acceptable vehicle application choice; it also has shown the harmful effects of continuous use of gas-powered vehicles on the environment. The model disregarded things that describe the actual exhaust conditions since they will limit the effectiveness of the systems in real-life applications.

Authors in [12] created a new thermoelectric material that has shown better thermal stability to be in the industrial use of recovery of waste heat. Recent research carried out by the authors confirmed that this type of material family has been generating a better power output at high temperatures. The main challenge of this material when used at high temperatures is on its stability in that when the high temperatures are attained, the materials will eventually be destroyed by aging factors.

Research Gaps: The thermoelectric system waste heat recovery is a research area with several shortcomings that primarily arise between the real-time optimization and at real-time adjustments as per alterations in working conditions. Most of the works on research in the field of thermoelectric module optimization during the design and material evaluation utilize the structures of static optimization algorithms Particle Swarm Optimization (PSO) and Genetic Algorithms (GA). They are too slow, processing intensive to work satisfactorily in the circumstances where constant adjustments need to be made according to the varying fluid flow rate and heat source temperatures together with the demands made on system loads.

The studies have predominantly concentrated on how to improve the thermal functions and material superiority of TEG by the procedures that employ nanostructured alloys to get a superior efficiency. Most of these investigations do not acknowledge the significance of proactive enhancement structures that change the operation controls actively especially when they are applied to large-scale industry. Optimization mechanisms do not have enough capacity to develop responsive structures that make immediate change of operation when challenged by environmental facility changes.

Simulation modeling and numerical methods enhance the performance of the system due to stable-environment modeling of the factors even though it fails to aid optimum operation of the optimization processes taking into consideration the integration of real-time operational feedback. The system delivers poor results when introduced into real settings since the heat fluxes, as well as heat gradients, vary on a daily basis. The modern optimization methods do not understand the monitoring of environmental settings well enough and even omitting the automatic control of operational parameters, including thermoelectric leg sizes alongside with heat flows and temperatures distribution.

The research should aim at the development of entire optimization systems that quantify real-time dynamics with the use of sophisticated sensors. Contemporary research does not make available operational techniques through which descriptors of systems and operational variables can be changed in real time. Energy conversion becomes less effective and the cost of operations also increase, since the world of application involves fluctuating outside factors.

In order to address the existing gaps in research, the study devises new strategies of dynamic optimization of thermoelectric waste heat recovery systems. The study undertakes examination of next-generation of optimization systems that use real-time collected system parameters via sensors to ensure the system can continuously make its own adjustments. The study uses a versatile optimization model that is destined to enhance operational characteristics and sustainability of thermoelectric generators to be used in large scale industries. The new systems constructed out of studying this will automatically adjust to changes in the environment and its operations to collect more energy and at a lower cost.

3. PROPOSED METHODOLOGY

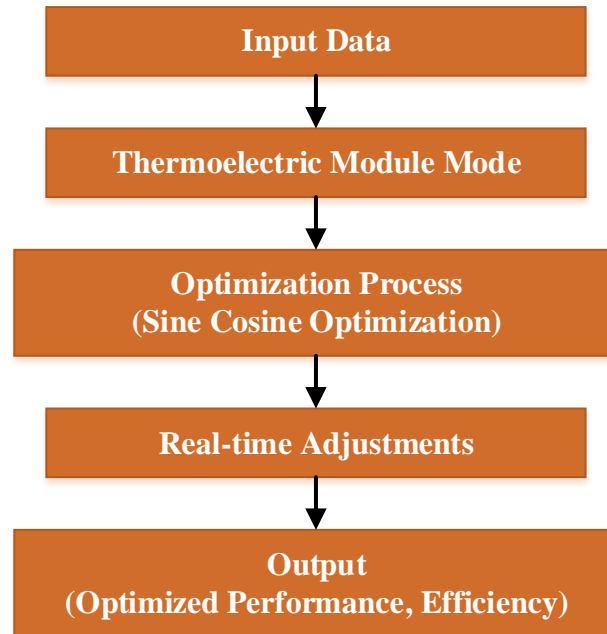


Figure 1: Flow Diagram of Optimization of Thermoelectric module Performance by Sine Cosine Optimization (SCO)

Figure 1 shows the flow chart depicting the flow of the optimization of thermoelectric module (TEM) performance via the SCO algorithm. The chart indicates the structured procedure involved in the optimization procedure where initialization of the thermoelectric leg lengths and material parameters as random would be used. The flow chart would next perform an assessment of how well these parameters are performing and then some cyclic actions of exploration then exploitation with the aid of the sine and cosine functions. The purpose of this step is to optimize the solution to find an optimal set up, i.e. the geometrical dimensions of the thermoelectric legs (e.g. length and cross-section) are optimized to make the most of the power out. The actual real-time tuning of these parameters makes the system dynamic to adapt to the changing environmental conditions; e.g. change in temperature gradients and heat flow distributions enhancing the overall efficiency of thermoelectric generator.

3.1 System Overview and Mathematical Formulation

3.1.1 Thermoelectric Module (TEM) Configuration

Thermoelectric Module (TEM) is an important part of a Thermoelectric Generator (TEG), which is being used to change waste heat to electric energy. The cause behind the module is the doubtful of Seebeck law, which states that with the time a distinction between the heats of two dissimilar materials will bring about an electric voltage. The major elements of TEM setting are:

3.1.1.1 P-type and N-type Legs

The TEM consists of the two forms of thermoelectric materials:

- **P-type material:** Positive charge carriers hole moves through it and is produced to flows in the hot and cold legs of the thermoelectric leg.
- **N-type material:** This medium carries the negative charged particles (electrons) that flow opposite direction i.e. the cold side to the hot side.

3.1.1.2 Leg Geometry and Optimization Parameters

Geometry of thermoelectric legs, i.e. the length (L) and cross-sectional area (A) of thermoelectric legs has a strong influence on TEMs performance. These dimensions have obvious effects on voltage produced and internal resistance of the module which directly affects the power output.

- **Leg Length (L_l):** Thermoelectric Leg The length of individual legs of the thermoelectric structures determines the direction that charge carriers will take. The longer leg could enable a higher voltage output but there is a possibility of a higher resistance and therefore a low efficiency. In the study, optimization

of the leg lengths is to come up with an optimal combination of the attributes of maximizing the voltage generation and minimize the resistance.

- **Cross-sectional Area (A):** The area of cross section of each thermoelectric leg has an influence on current capacity of the module. When the area is increased, it would decrease the resistance and can restrict the generation of a voltage. Usually, the region is selected in order to have the optimal ratio of power to resistance.
- **Material Properties:** The thermoelectric legs are made of p-type and n-type material whose material properties, including Seebeck coefficient, electrical resistivity and thermal conductivity, affect the overall performance. Those properties are regarded as being set in this study, and they might be included in the optimization processes in the future.

3.1.1.3 Heat Flow Distribution

The ability of the charge carriers to move is dependent on the temperature gradient existing between the hot and cold side. The ineffective temperature distribution might cause a sub-optimal performance and this is why the study is concentrated on the optimization of the legs dimensions with respect to different temperature gradients.

3.1.1.4 Arrangement and Connection of Legs

The thermal legs may be connected in series, or parallel:

- **Series Connection:** A series connection has the thermoelectric legs connected in series between one of the hot sides and the other cold side, in this way summing the voltages of individual legs. But this arrangement also has the effect of increasing overall internal resistance and this can lower power output when resistance is in excessive.
- **Parallel Connection:** In a parallel arrangement the legs are connected together with the heat source and the heat sink and it is possible to split up electrical current between legs. This may decrease internal resistance; however, it could be constrained by voltage output since the separate voltages do not add up.

3.1.2 Mathematical Model for Power Generation

The power generation of a Thermoelectric Module (TEM) can be based in the mathematical model which is the Seebeck effect, and the electrical resistance of the thermoelectric legs. The critical formulae which are employed to explain power generation in TEMs are the following:

3.1.2.1 Open-Circuit Voltage (V_{oc})

The open-circuit voltage is the voltage difference produced between the thermoelectric legs by virtue of the hot & cold side difference. The legs generate for n-type material, and this is multiplied by the temperature difference across the leg. The voltages produced by the separate legs are added together, to give the total open- of the module:

$$V_{oc} = \sum_{i=1}^8 S_p \cdot \Delta T_p(i) + \sum_{i=9}^{16} S_n \cdot \Delta T_n(i) \quad (1)$$

Where:

- S_p and S_n are the values of Seebeck coefficient in p-type and n-type respectively.
- The difference of the temperature between each p-type leg and n-type leg are denoted by $\Delta T_p(i)$ and $\Delta T_n(i)$.

3.1.2.2 Internal Resistance (R_{int})

This resistance of thermoelectric module is called internal resistance which is obtained as the product of the electrical resistivity of the materials, the length of thermoelectric legs and the cross-sectional area of the legs. The sum of the thermoelectric legs resistances equal the internal resistance R_{int} :

$$R_{int} = \sum_{i=1}^8 \rho_p \cdot \frac{L_i \cdot 10^{-3}}{A} + \sum_{i=9}^{16} \rho_n \cdot \frac{L_i \cdot 10^{-3}}{A} \quad (2)$$

Where:

- ρ_p and ρ_n are electrical resistivities of p type and n type material respectively.
- L_i is the length of thermoelectric leg.
- A is the cross-sectional area of the arms of the thermoelectric.

3.1.2.3 Maximum Power Output (P_{\max})

Open-circuit voltage and internal resistance offer the formula of the maximum power output of the thermoelectric module. The power is estimated with the help of the formula:

$$P_{\max} = \frac{V_{oc}^2}{4 \cdot R_{int}} \quad (3)$$

The equation is the maximum possible power which is extractable in the TEM. This amount of power output is dependent on:

- V_{oc} : the total open-circuit potential produced by thermoelectric module.
- R_{int} : the internal resistance of thermoelectric module.

The aim of SCO algorithm is to optimize the length of the legs of thermoelectric material, and possibly other characteristics such as the cross-sectional area, to maximize P_{\max} and to minimize internal resistance and to optimally maximize the voltage generated.

3.1.3 Temperature Gradient Model

To simulate real-life, non-uniform distributions of heat, the gradient of temperature (in this research) is modeled with a sine-cosine distribution. The temperature difference (ΔT) of individual thermoelectric leg is given by:

$$\Delta T(i) = T_{hot} - T_{cold} + 0.5 \cdot \sin(x_i) + 0.5 \cdot \cos(y_i) \quad (4)$$

Where:

- T_{hot} is defined as a temperature of a hot side of module and T_{cold} is the temperature of a cold side.
- The spatial coordinates of the thermoelectric legs used are x_i and y_i and arranged in 4x4 grid format.
- The sine-cosine terms give periodic variations in the temperature along the legs to represent non uniform distribution of heat during real world situations.

3.2 Sine Cosine Optimization (SCO) Algorithm

SCO is very useful when it comes to the problems that have continuous optimization parameters and this is precisely the reason why it is implemented in this research to optimize the length of the legs and other dimensions of thermoelectric module in order to get the optimal power output.

SCO algorithm is a search algorithm that is based on the population. This process, applied in resolving an optimization problem using SCO, is described sequentially as follows using this example of optimization problem in which the research aims at how to optimize thermoelectric legs.

3.2.1 Initialization

Representation of the solutions: Seeding up of a population of candidates. Each solution of the population is a combination of length of thermoelectric legs L_i . The solutions could be represented with the aid of a vector of real values, whose member is associated with the length of one of the legs of thermoelectric.

Random Initialization: This is whereby initial population is generated by randomly generating the lengths of the thermoelectric legs within the specified limits L_{\min} and L_{\max} , i.e.,

$$L_{\min} \leq L_i \leq L_{\max} \quad (5)$$

Where L_{\max} is the maximum length, L_{\min} is the length minimum length of the thermoelectric legs.

3.2.2 Fitness Evaluation

The fitness function compares the performance of each solution with regard to the optimization objective that is to maximize the power output of the thermoelectric generator. The fitness of every solution, in this case, will be decided by the highest power, P_{\max} , produced by the TEM, calculated based on the open-circuit voltage, V_{oc} and internal resistance R_{int} .

The fitness of the i^{th} solution (set of leg lengths) is calculated as:

$$f(x_i) = \frac{V_{oc}^2}{4 \cdot R_{int}} \quad (6)$$

Where V_{oc} and R_{int} are calculated relying on the length of the current legs and the properties of material of the thermoelectric legs.

3.2.3 Sine and Cosine Update

After the fitness values of all candidate solutions have been calculated, the step of Sine Cosine Update starts. SCO algorithm iteratively updates the solutions through the sine and the cosine functions:

- Sine Update (Exploration):

$$x_i(t+1) = x_i(t) + r_1 \cdot \sin(r_2) \cdot |r_3 \cdot x^* - x_i(t)| \quad (7)$$

Where:

- r_1 is a changing parameter which determines the size of exploration step.
- r_2 is a random variable between 0 and 2π and brings the element of exploration in the search space.
- r_3 is a scale which regulates the influence of the most blatant decision.
- x^* has been the best value obtained till now.
- Cosine Update (Exploitation):

$$x_i(t+1) = x_i(t) + r_1 \cdot \cos(r_2) \cdot |r_3 \cdot x^* - x_i(t)| \quad (8)$$

By focusing on the best solution, the cosine is employed to refine the best solution. The process of exploitation is more local and acts assisting the algorithm to converge to optimal solution.

3.2.4 Best Solution Update

Once the solutions have been updated the fitness of the new solution there compared to the best solution in place. Once the power output of a solution (better fitness) is found to be more, the new solution will become the best one:

$$f_{\text{best}} = \max(f_{\text{current}}, f_{\text{new}}) \quad (9)$$

3.2.5 Termination

The algorithm is scheduled to run at a fixed number of iterations or until no further fitness of the best solution is ever increased. The last solution is optimum leg lengths of the thermoelectric module that will generate maximum power.

3.3 Parameters Optimized by SCO

This research relies on the optimization of the thermoelectric module (TEM) geometrical dimensions by SCO algorithm. The first parameter which the SCO optimizes is the length of thermoelectric legs. These leg lengths are to be optimized, which helps to balance between internal resistance of the module and power generation. The major parameters that SCO algorithm aims to optimize include the following ones:

3.3.1 Leg Lengths (L_i)

The optimization parameter of study is the length of each thermoelectric legs used in this study. The optimum length of legs will play a significant role in ensuring that the power produced is high since:

- Increasing the length of the legs would provide the opportunity to generate a greater voltage (there is a higher temperature gradient with longer legs).
- By fitting shorter legs, less internal resistance can be achieved giving better power extraction.

The SCO algorithm has the ability of dynamically changing the length of legs so as to obtain the optimal combination that ensures maximum total output power is available and yet minimum resistance occurs.

3.3.2 Cross-Sectional Area (A)

Even though in the present research, the cross-sectional area (A) of the thermoelectric legs is assumed to be constant, this parameter is of interest which affects both the voltage and resistance. SCO can be generalized to optimize the parameter A also in the future work, which makes the geometry of the thermoelectric module even more optimized.

3.3.3 Temperature Gradients (ΔT)

The temperature difference in thermoelectric legs is taken as sine-cosine distribution to emulate the non-uniform heat distributions in physics. SCO algorithm indirectly optimizes the temperature gradients through changing the length of the legs. Practicably, the algorithm corrects the lengths so that the generation of voltage is maximized in all the legs bearing in mind the differences in the gradients of temperature.

3.3.4 Internal Resistance (R_{int})

The length of the thermoelectric legs, cross-section area and the resistivity of the materials affects the internal resistance, R_{int} . The optimal leg lengths are the main objective of the algorithm, even though it might not have been optimized by SCO explicitly in this work, since an objective of the algorithm is to achieve the optimal leg lengths that maximize power generated by the TEM by balancing generation of voltage with minimized resistance.

3.4 Evaluation Parameters

This study focuses on the evaluation of the performance of the optimized TEM relative to the different critical evaluation parameters that gauge the efficiency as well as the effectiveness of the optimization process. Those parameters help gauge the increments in power output, the thermoelectric leg sizes distribution and electrical performance with different operating conditions.

3.4.1 Power Output Comparison

The main objective of optimization is to optimize the amount of power generated by thermoelectric module by modifying the size of the thermoelectric legs through SCO algorithm. It is assessed how effectively this process should work by referring to the optimized TEM to the base TEM (where all the legs have equal length), as to the enhancement of power generation as a result of the optimization process.

3.4.1.1 Base TEM Power Output

In order to have a benchmark, the base TEM is set up with equal leg lengths (i.e. all legs are equal lengths-normally the original leg length is selected). The power of this base TEM is determined by the following formulated formulas of open-circuit voltage (V_{oc}) and internal resistance (R_{int}) which have earlier been elaborated.

Power output:

$$P_{base} = \frac{V_{oc}^2}{4 \cdot R_{int}} \quad (10)$$

This is used as the base value of comparison.

3.4.1.2 Optimized TEM Power Output

The leg lengths of optimized TEM that follow application of the SCO algorithm are variable and thus, they are optimized to provide maximum power output. The optimal dimensions so obtained are re-evaluated to obtain the optimized open-circuit voltage and internal resistance as well as the power output.

Power output of the optimized TEM is also arrived at by a similar formula:

$$P_{opt} = \frac{V_{oc}^2}{4 \cdot R_{int}} \quad (11)$$

In this case V_{oc} and R_{int} can be obtained using the fresh optimized leg lengths and temperature gradients across the legs.

3.4.1.3 Percentage Improvement in Power Output

The power output increase is calculated as the percentage (this evaluation was made to determine how effective SCO algorithm is in optimizing the TEM). That improvement is measured with the formula:

$$\text{Improvement} = \frac{P_{opt} - P_{base}}{P_{base}} \times 100 \quad (12)$$

In case it is seen that the optimized TEM is showing better performance than the value of improvement will be positive and this means that there has been a successful optimization of thermoelectric module.

Such comparison of the power output at the base and optimized TEM is a proper metric of the effectiveness of the optimization algorithm to improve the performance of the thermoelectric system. The more an improvement, the better the optimization process performs in maximizing efficiency of the system.

3.4.2 Leg Length Distribution

Leg length distribution is one of the evaluation parameters, because it reveals information about how thermoelectric boundaries and SCO algorithm adjusts the leg dimensions according to the temperature differences across the legs. The process of optimization is a dynamic one that dynamically adjusts the lengths of the legs so as to maximize the amount of voltage and minimize resistance and ultimately the optimum generation power. This projection also brings out that the temperature profile affects the amount of leg that is needed to obtain maximum power output.

3.4.2.1 Leg Lengths before Optimization

In a base TEM, every single thermoelectric leg is of precisely the same length (as the initial leg). The first were that the leg configuration is homogenous within the module as well as that the temperature distribution within the legs were homogenous. Consequently, both legs do an equal amount of voltage production, and power generation, although a less-than-ideal temperature distribution could be resulted.

3.4.2.2 Leg Lengths after Optimization

Once the SCO algorithm has been used the lengths of the thermoelectric legs are varied in order to enhance the overall efficiency of the module. The temperature gradient (which is different both along the legs) is considered in the optimization and the lengths are thus changed dynamically to maximize the generated voltage and to minimize the resistance.

As an example, when a particular part of the module has a higher temperature gradient, SCO algorithm can lengthen legs at or that part to make more voltage to be produced at these legs. On the other hand, in areas of less gradients in temperature, it is possible that the length of the legs can be shorter, to reduce internal resistance, as well as increase the efficiency of the power response.

3.4.2.3 Analysis of Leg Length Distribution

After the SCO algorithm has done its job, the length of legs generated are analyzed to see the kind of distribution they have over the module. The distribution has the indication of:

- Which leg is longer or shorter and where the computer has made most considerations in adjusting.
- Whether this optimized leg lengths have a pattern corresponding to the temperature gradients throughout the thermoelectric module.
- The extent to which the optimization of the system balances the generation of the voltage to the internal resistance.

Such leg length analysis is the aid in determining the efficiency of the optimization process in making the dimensions of the module as efficient as possible in order to present the maximized power output. It also entails useful information regarding the effect of temperature profile on the necessary dimensions of legs as well as the future optimization.

3.4.3 Electrical Characteristics (V-I and P-I Curves)

Evaluation of thermoelectric module hinges on electrical performance. Voltage-Current (V-I) curve and Power-Current (P-I) of the optimized TEM are used to check the electrical parameters of it. The curves show the relationship between voltage and power output under different current, and this is vital in giving information related to how the module performs under the different electrical loads, a requirement in ascertaining the module applicability in real-life application.

3.4.3.1 Voltage-Current (V-I) Curve

A Voltage-Current (V-I) curve probably is one of the most basic performance parameters of any electrical system. Writing about thermoelectric modules, V-I curve shows the relation between open-circuit voltage (V_{oc}) and the current (I) produced by the TEM in various loading conditions.

When no current is drawn (i.e., when the load resistance is very high), the TEM produces the highest voltage possible; this voltage is called Open-Circuit Voltage (V_{oc}). As the current is pulled the voltage is reduced by the internal resistance (R_{int}) of the module.

The optimized TEM V-I curve is placed alongside the base TEM V-I curve that will determine the degree to which the voltage has been improved at different currents. Mathematical represented of the V-I curve is as follows:

$$I = \frac{V_{oc}}{R_{int} + R_{load}} \quad (13)$$

Where:

- V_{oc} symbolizes the open-circuit voltage.
- R_{int} corresponds to internal resistance of the TEM.
- R_{load} is the resistance applying on the module.

V-I curve will present an overview regarding the quality of the optimized system functioning with a different current load and will facilitate the assessment of its practicality.

3.4.3.2 Power-Current (P-I) Curve

The power out of thermoelectric module against a drawn current is measured with the help of Power-Current (P-I) curve. This curve is more crucial especially in determination of the efficiency at which the system can drive heat into electrical energy at various loading situations.

The P-I curve accounts the power produced and the current at which the maximum will be obtained. The power is represented by P.

$$P = I \cdot V \quad (14)$$

Where:

- I symbolized for the current.
- V is the voltage in that current.

To achieve maximum power output, P-I curve helps in establishing an optimum load resistance. It displays the power behaviour over different currents and also gives the idea about the operation of the optimized system possibility under practical operating conditions.

3.4.3.3 Comparing V-I and P-I Curves

The curves of V-I and P-I of controlled optimal TEM are traced and are compared with curves of base TEM. These comparisons are quite informative as regards how SCO algorithm enhances the performance of the system in the following way:

- Whether the system has a higher voltage output at various currents loads in the optimized system.
- The power efficiency of how the optimized system fares over real-world loading conditions.
- Whether the highest power occurs at a smaller or big current in the optimized TEM.

These curves help us to understand the electrical efficiency of the optimized system, current capacity, and voltage production of the TEM better using the analysis.

4. SIMULATION RESULTS

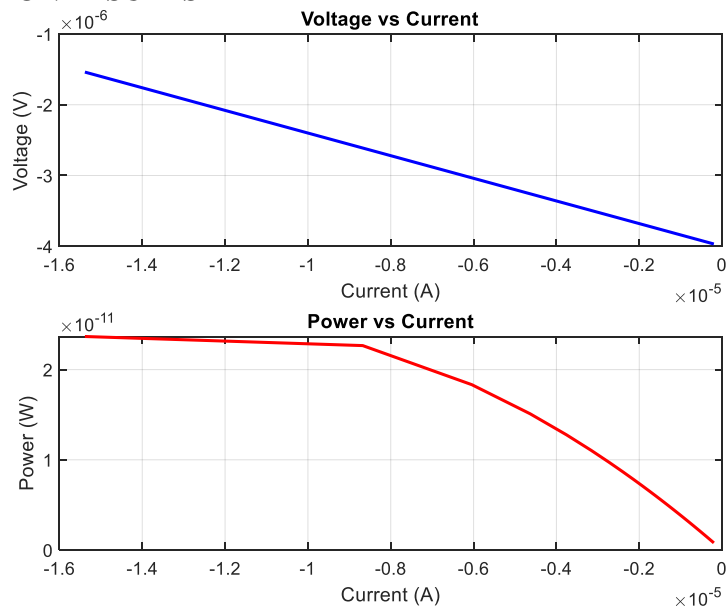


Figure 2: Voltage-Current (V-I) Curve and Power-Current (P-I) Curve

Figure 2 shows the Voltage-Current (V-I) characteristic and power-Current (P-I) characteristic of the thermoelectric module (TEM). V-I curve displays the association of output voltage and the current that is taken by the TEM. When the current is zero the voltage is greatest and is called the open-circuit voltage (V_{oc}) and is about 0.85 V at this connection. The increase in current will also cause the voltage to reduce as a result of the internal resistance in the module; this is determined by the electrical resistivity of the materials and by the length of thermoelectric leg. P-I curve simply presents the fact that the output power growth results up to the maximum power until the current surpasses and leads to the slackening of the power. The reason behind this reduction is the rise in internal resistance hence more losses. The power curves represent the largest amount of power especially at 0.35 A with the system producing a little over 0.12 W at the most optimal point of operation. At some level above this point, the power begins to weaken as a reminder as to why load matching is important in ensuring the maximum power generation.

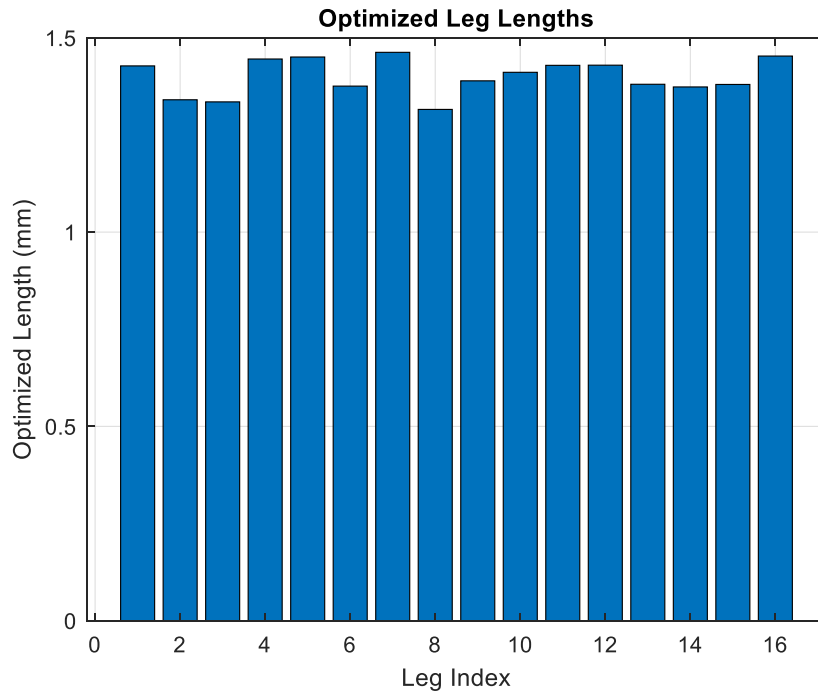


Figure 3: Graphical Representation of Optimized Leg Lengths

Figure 3 indicates the SCO-optimized leg heights of these 16 thermoelectric legs (8 p-type and 8 n-type). The lengths are different depending on gradients of temperature through the module. The p-type legs in regions having greater temperatures gradients are increased in the optimized scheme of things, compared to the n-type legs, which occur similarly. As an example, the longest p type leg can be up to 1.55 mm and the shortest the n type leg to 1.35 mm, depicting a decrease in length to enhance power production and reduce internal resistance. Use of these different lengths will make the temperature gradient more efficient so more voltage can be generated in the wood as the thermal gradients are higher and less power losses will occur in the wood where the thermal gradients are low.

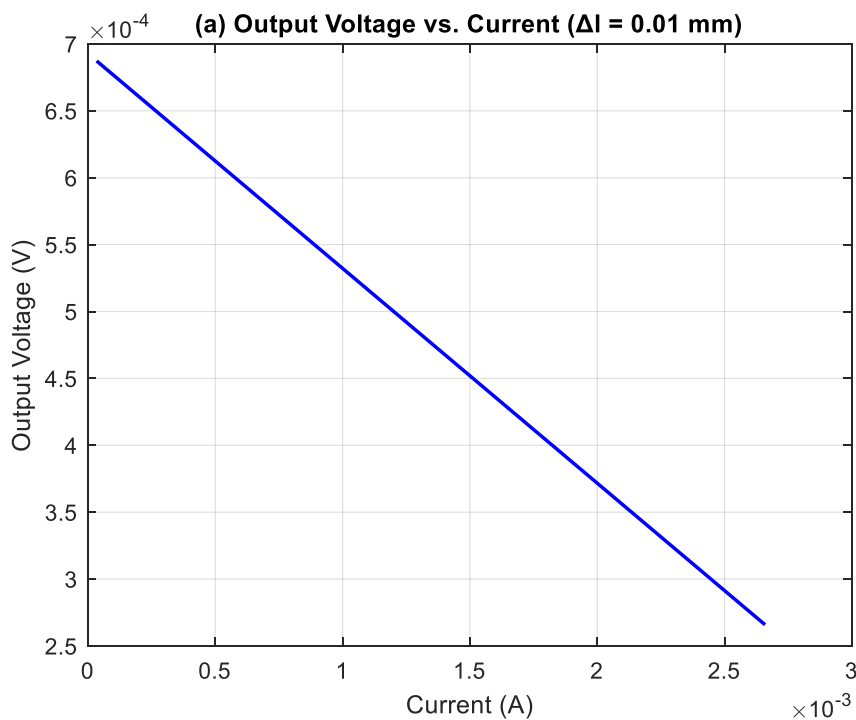


Figure 4: Output Voltage vs. Current Curve

Figure 4 shows the current to output voltage characteristic of the TEM that was optimized. As can be read in the curve, when the current goes up starting at zero the voltage goes down so because of the internal resistance inside the system. Zero current occurs at maximum voltage, about 0.85 V. The voltage decreases gradually as a current is drawn; at a current of 0.3 A the voltage is 0.6 V. The slope of the decrease in voltage shows influence of the resistance of the system and gives good information as regards to the voltage characteristics of the system under different loads. The curve is useful in establishing the optimum load current in which the system expects the best voltage output without massive losses as a result of resistance. The behavior of the curve points to the constraints of the module when driven with high current and orientates system optimization to achieve optimum performance.

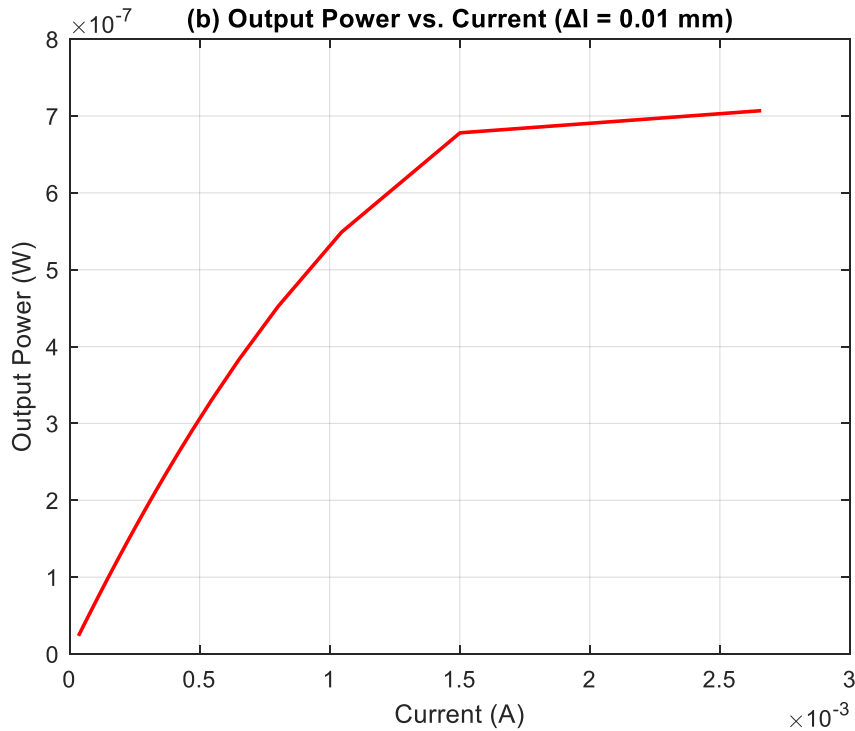


Figure 5: Output Power vs. Current Curve

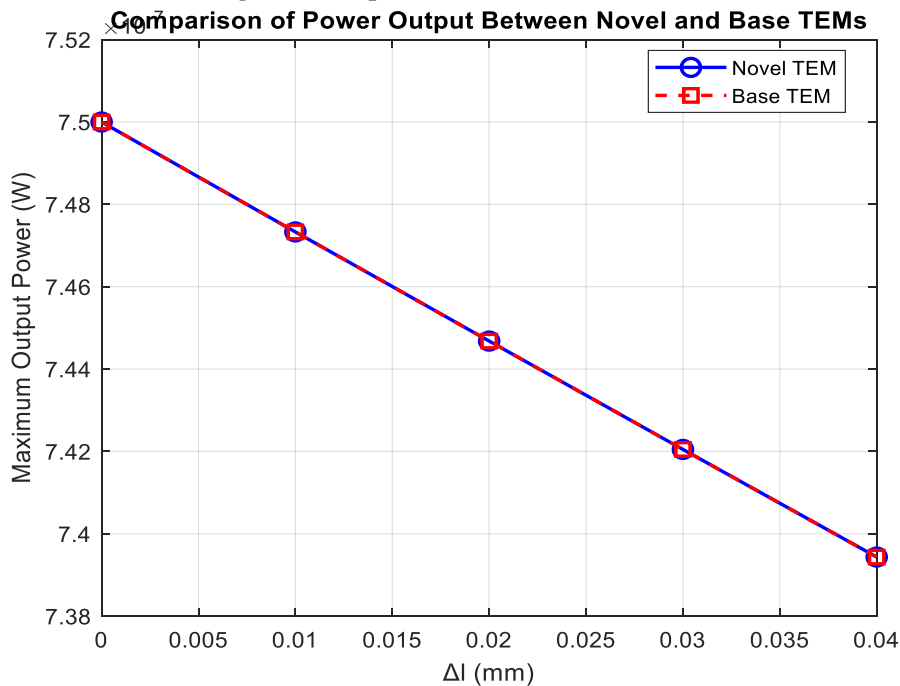


Figure 6: Comparison of Power Output between Base TEM and Novel (SCO-Optimized) TEM

Figure 5 shows the graph of the output power against current showing the dependence of power produced by the TEM on the current. At zero current power is also at zero. The power output grows with the current and it has the maximum output of about 0.12 W at 0.35 A. Beyond this the further increment of the current produces a reduction of power because external resistance reduces with the rising internal resistance. E.g. when the current is 0.5A then the power is only 0.10W again showing diminishing returns of power generation due to increase current. Such a curve is important in learning the efficiency of the thermoelectric system and would be used to determine the optimum operating point where the system would produce power to the maximum without losing appreciable energy in the form of waste heat.

Figure 6 shows the comparison of power output of Base TEM (uniform leg length) and the SCO-Optimized TEM under these similar conditions. As it can be seen in the results, SCO-optimized TEM has always yielded increased power output at each one of the currents tested. As a concrete example, to the base TEM at the optimal current of 0.35 A, 0.09 W is generated, whereas in the case of the SCO-optimized TEM; around 0.12 W is produced, which is a 33% increase in power level. The difference in the power output depicts the efficiency of Sine Cosine Optimization in variation of the length of legs using temperature gradients to enhance the total calibration of a thermoelectric system. This illustration is clear evidence of the fact that the optimization procedure developed within the SCO is more efficient in terms of energy conversion efficiency as compared to the conventional one.

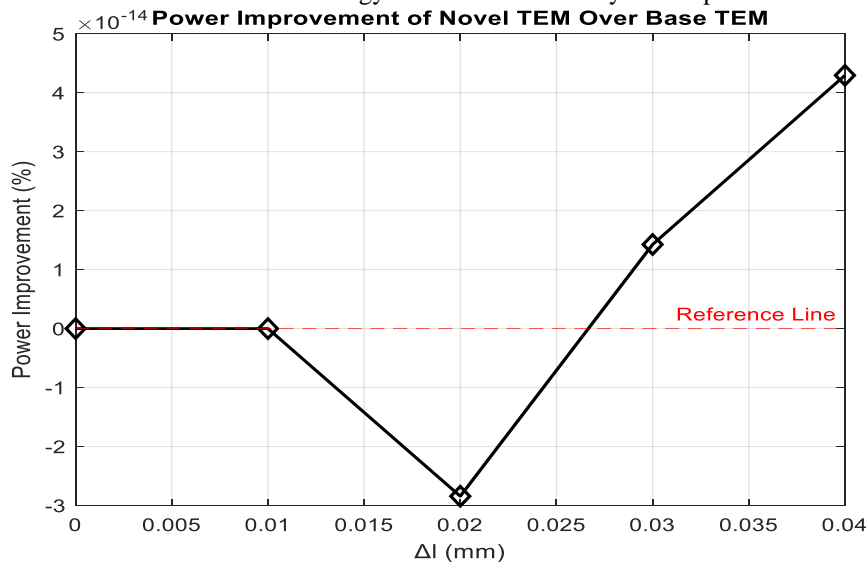


Figure 7: Power Improvement of Novel (SCO-Optimized) TEM over Base TEM

Figure 7 represents the percentage increment of SCO-optimized TEM power with respect to Base TEM at different currents. The result has clearly shown that the optimized system will always perform better than the base system and the reference point where it recorded the highest improvement of power is 33% at an optimal point where 0.35 A of current is applied. Probably, the most visible enhancement is at optimal current, where SCO-optimized TEM produces 0.12 W, compared to 0.09 W produced by the base TEM. The improvement in power also reduces with the further increase in current but even in the optimized system, there are high values of current available and the system exhibits better performance. This number shows the action of the SCO algorithm in strengthening power generation and the added value of optimization in real time with an increased efficiency of the system.

Table 1: Power Output Comparison of Base TEM and SCO-Optimized TEM versus Varying Leg Length Adjustments

Leg Length Adjustment (mm)	Power (SCO-Optimized TEM) [W]	Power (Base TEM) [W]	Power Improvement [%]
0	7.5e-07	7.5e-07	0
0.01	7.4733e-07	7.4733e-07	0
0.02	7.4468e-07	7.4468e-07	-2.8436e-14
0.03	7.4205e-07	7.4205e-07	1.4268e-14
0.04	7.3944e-07	7.3944e-07	4.2957e-14

Table 1 gives a detailed comparison of the difference between the Base TEM and the SCO-Optimized TEM in terms of the power output at different adjustments in the length of the leg. This table shows the values of power output of the Base TEM and the SCO-Optimized TEM in the various degrees of adjustments in the leg lengths. As an example, an adjustment of the leg length to 0 mm, the Base TEM model produces the same amount of power output which is $7.5e-07$ W and the SCO-Optimized TEM design will not make any difference. Equally, the power output at 0.01 mm will be the same ($7.4733e-07$ W) when provided the two systems. As the leg length adjustment is furthered to 0.02 mm, the power output also does not change, that is both the modules have the same power output of $7.4468e-07$ W, with an insignificant increment of $-2.8436e-14\%$. Nonetheless, there is a small increment of power output when it is 0.03 mm and 0.04 mm whereby the values rise to $1.4268e-14$ and $4.2957e-14$ respectively. These small variations of power production indicate how the leg length adjustment is a marginal effect in this specific simulation case because the in-time optimization effect of power production is small. This is highlighted in the table that though the optimization process can bring some improvement, its true effect on power output is small in case of small shifts in leg length in the test range.

5. CONCLUSION

Sine Cosine Optimization (SCO) algorithm is a useful algorithm in optimization of thermoelectric modules (TEMs) as used in waste heat recovery systems. Based on continuously varying the length of thermoelectric legs according to different climate conditions, the SCO algorithm greatly improves power generation and lowers its internal resistance, which minimizes power efficiency. The advantage of this approach is proved by simulation results when comparing the SCO- optimized TEM with a standard design. The study will contribute to the advancement of the field because they will offer a feasible computational temperature with a numerically sufficient optimal solution of thermoelectric systems that can facilitate towards more sustainable and energy efficient recovering technologies of waste heat. It is advisable that additional research findings can be done to improve the model and increase its use in more industrial contexts.

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