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COMPRESSION OF NATURAL IMAGES USING WAVELET TRANSFORM

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Abstract

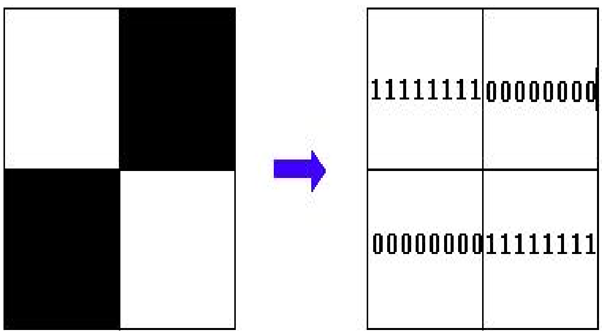
This work proposes a new approach to increase the quality of natural images by compressing the images into large extent. The proposed scheme is used on different types of families of wavelets and experiments on Matlab are done using wavelet transform. Different compression algorithms are also applied for best results. Quality estimation is done on the basis of entropy calculation. However other parameters like compression ratio, peak signal to noise ratio and energy retained are also calculated for comparison these wavelet families.

Key Words: Image compression, PSNR, energy retained, threshold, entropy, Db, Symlet, Biorthogonal and coiflet wavelet

1. Introduction

Image compression technique is required to minimize the amount of memory needed to represent an image. Images often require a large number of bits, and if the image needs to be transmitted or stored, it is not possible to do so without reducing the number of bits. The problem arises when we transmit or store large images. TV and fax machines are both the examples of image transmission, and digital video players are the examples of image compression [1] [2] [3] [4] [5] [6] [7][8].

Each level of an image is represented by an 8- bit binary number, such that black is represented by 00000000 and white is 11111111. An image can therefore be thought of as grid of pixels, where each pixel can be represented by the 8-bit binary value for grey scale. The resolution of image is given by pixels per square inch or dots per inch (dpi). So 500dpi means that a pixel is 1/500th of an inch.To digitize a one-inch square image at 500dpi requires 8*8*500=2 million storage bits. So image data compression is a great advantage if image is required to be stored, transmitted and processed.



2. Methods of Compression

Fourier Analysis:. Fourier analysis is a mathematical function for transforming signal from time based to frequency based. It breaks down a signal in to constituent sinusoids of different frequencies. But, In transforming to the frequency domain, time information is lost. During Fourier transforms of a signal, it is impossible to tell when a particular event took place. If the signal properties do not change much over time it is called a stationary signal. [13].

JPEG Compression: - JPEG stands for the Joint Photographic Experts Group, a standards committee that had its origins within the International Standard Organization (ISO). JPEG provides a good compression method that is capable of compressing continuous-tone image data with a pixel depth of 6 to 24 bits with reasonable speed and efficiency. And although JPEG itself does not define a standard image file format, several have been invented or modified to fill the needs of JPEG data storage [14] [15].**Wavelet Analysis:-**The signal is defined by a function of one variable or many variables. Any function is represented with the help of basis function. An impulse is used as basis function in time domain can be represented in time as a summation of various scaled and shifted impulses. Sine function is used as the basis in the frequency domain. But, these two basic functions have their own weakness: an impulse is not localized. In the frequency domain and thus a poor basis function to represent the frequency information. In order to represent complex signal efficiently, a basis function should be localized in both time and frequency domain. The support of such basis function should be variable, so that narrow version of function can be used to represent the high frequency components of a signal while wide version of function can be used to represent the low frequency components. Wavelets satisfy the condition to be qualified as the basis function.

One of the most commonly use approaches for analyzing a signal f(x) is to represent it as a sum of simple building blocks, called basis function.

$$f(x)=\sum C_i\Psi_i(x)$$

Where the$\Psi_i(x)$ are basis function and C_i are coefficients, or weights. Since the basis functions Ψ_i are the fixed, it is the coefficients which contain the information about the signal. The simplest such representation uses translate of the impulse function as its only bases, yielding a representation had reveals information only about the time domain behaviour of the signal. Choosing the sinusoidal as the basis functions yields a Fourier representation that reveals information only about the signal‘s frequency domain behaviour. Wavelets are function that satisfy certain mathematical requirements and are used in representing data or other functions. The basic idea of wavelet transform is to represent any arbitrary signals X as a superposition of such wavelets or basis functions. These basis function are obtained from a single photo type wavelet called mother wavelet by (scaling) and translation(shifting).For the purpose of signals compression the representations is ideal about both the time and frequency behaviour of signal. Resolution in time (Δx) and resolution in frequency ($\Delta\omega$) cannot both the made arbitrarily small at the same time because their product is lower bounded by the Heisenberg inequality.

$$\Delta x \, \Delta\omega \geq 1/2$$

This inequality means that we must trade off time resolution for frequency resolution, or vice versa. Thus it is possible to get very good resolution in time to settle for low resolution in frequency and you can get very good in frequency to settle for low resolution in time [16].

3. Types of Wavelets

Daubechies Maxflat Wavelet: - There are many wavelets available to decompose and analyze both discrete and continuous data. Harr filter represents special case of Daubechies filter family. Harr filter is actually Daubechies filter of order 1.The construction is based on solving the frequency response function for the filter coefficients satisfying orthogonality and moment conditions. The main feature of Daubachies family is orthogonailty and asymmetry. The support length of scaling and wavelet function is 2N-1.the number of vanishing moment of wavelet function is N. Filter length is 2N[17].

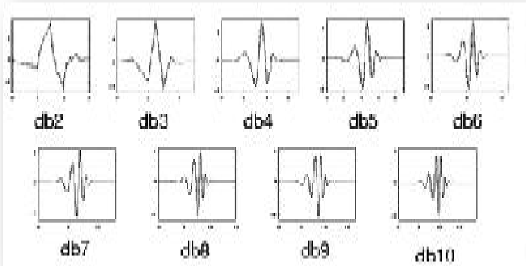


Figure 1 Representation of Daubachies Wavelet

Coiflet wavelet: - Coiflet is a discrete wavelet designed by Ingrid Daubechies to be more symmetrical than the Daubechies wavelet. Whereas Daubechies wavelets have $N / 2 - 1$ vanishing moments, Coiflet scaling functions have $N / 3 - 1$ zero moments and their wavelet functions have N / 3.

Coefficients:- Both the scaling function (low-pass filter) and the wavelet function (High-Pass Filter) must be normalized by a factor. Below are the coefficients for the scaling functions for C6-30. The wavelet coefficients are derived by reversing the order of the scaling function coefficients and then reversing the sign of every second one.

Scaling function: - As in the orthogonal case, □(t) and □(t/2) are related by a scaling equation which is a consequence of the inclusions of the resolution spaces from coarse to fine:

$$\frac{1}{\sqrt{2}}\phi\left(\frac{t}{2}\right) = \sum_{n=-\infty}^{+\infty} g[n]\,\phi(t-n),$$

Similar equations exist for the dual functions which determine the filters h_2 and g2.

Vanishing moments: - A coiflet wavelet has m vanishing moments if and only if its *dual* scaling function generates polynomials up to degree m. Hence there is an equivalence theorem between vanishing moments and the number of zeroes of the filter's transfer, provided that duality has to be taken into account. Duality appears naturally, because the filters determine the degree of the polynomials which can be generated by the scaling function, and this degree is equal to the number of vanishing moments of the *dual* wavelet. certain number of vanishing moments on a scaling function (e.g., coiflets) leads to fairly small phase distortion on its associated filter.real-valued, compactly supported, orthonormal, and nearly symmetric wavelets (we call them generalized coiflets) with a number of nonzero-centered vanishing moments equally distributed on scaling function and wavelet. Such a generalization of the original coiflets offers one more free parameter, the mean of the scaling function, in designing filter.

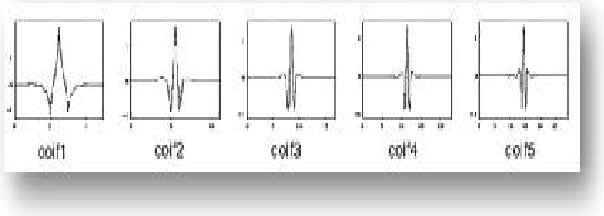


Figure 2 Coiflet filter of different order

Bi-orthogonal Wavelet: - A biorthogonal wavelet is a wavelet where the associated wavelet transform is invertible but not necessarily orthogonal. In the biorthogonal case, there are two scaling functions $\phi, \tilde{\phi}$, which may generate different multiresolution analyses, and accordingly two different wavelet functions $\psi, \tilde{\psi}$. So the numbers M, N of coefficients in the scaling sequences a,ã may differ. The scaling sequences must satisfy the following biorthogonality condition[18]

$$\sum_{n\in\mathbb{Z}} a_n \tilde{a}_{n+2m} = 2 \cdot \delta_{m,0}$$

Then the wavelet sequences can be determined as, $b_n = (-1)^n \tilde{a}_{M-1-n}$ n=0,...,M-1

and, $\tilde{b}_n = (-1)^n a_{M-1-n}$ n=0,....,N-1.

Scaling equation: As in the orthogonal case, y(t) and j(t/2) are related by a scaling equation which is a consequence of the inclusions of the resolution spaces from coarse to fine:

$$\frac{1}{\sqrt{2}}\psi\left(\frac{t}{2}\right) = \sum_{n=-\infty}^{+\infty} g[n]\,\phi(t-n),$$

Similar equations exist for the dual functions which determine the filters h2 and g2.

Vanishing moments: A biorthogonal wavelet has m vanishing moments if and only if its dual scaling function generates polynomials up to degree m. Duality appears naturally, because the filters determine the degree of the polynomials which can be generated by the scaling function, and this degree is equal to the number of vanishing moments of the dual wavelet

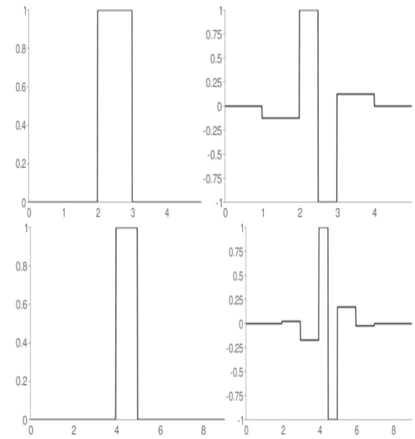


Figure 3 Bi-Orthogonal wavelet filter order of (1.3,1.5)

Symlet wavelet: - Symlets are also orthogonal and compactly supported wavelets, which are proposed by I. Daubechies as modifications to the db family. Symlets are near symmetric and have the least asymmetry. The associated scaling filters are near linear-phase filters. Daubechies, Symlet and Coiflet filters having special property of more energy conservation, more vanishing moments, regularity and asymmetry than other biorthogonal filters. For example, in the case of Daubechies wavelets we have a maximum number of vanishing moments and maximal asymmetry with fixed length of support, while the Symlet wavelet family has the "least asymmetry" and highest number of vanishing moments with a given support width.

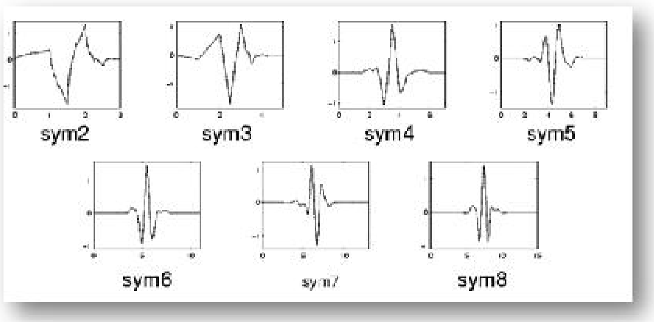


Figure 4 of Symlet wavelet filters of different order

4. Wavelet Filter Selection

Discrete Wavelet Transform (DWT) is a popular technique for image coding applications. In this method the entire image is transformed and compressed as a single data object rather than block to block, allowing for a uniform distribution of compression error across the entire image. The blocking artifacts and mosquito noise are absent in a wavelet based coder due to the overlapping basis functions [19]. These wavelet functions can be divided into two parts: orthogonal and biorthogonal. Orthogonal wavelets use the similar filter for reconstruction whereas the length of reconstruction filter differs from the synthesis filter in case of biorthogonal wavelets. The selection of wavelet function is crucial for performance in image compression [8]. Important properties of wavelet functions in image compression applications are compact support, symmetry, orthogonality, regularity and degree of smoothness [9] [10]. There are a number of basis that decides the selection of wavelet for image compression. Since the wavelet produces all wavelet functions used in the transformation through translation and scaling, it determines the characteristics of the resulting wavelet transform. Therefore, the details of the particular application should be taken into account and the appropriate wavelet should be selected in order to use the wavelet transform effectively for image compression.

Daubechies Wavelet (DB), Biorthogonal Wavelet (BIOR), Coiflet Wavelet (COIF) and Symlet (SYM) are analyzed in literature [6]. The DB, BIOR, and COIF wavelets are families of orthogonal wavelets that are compactly supported. These wavelets are capable of perfect reconstruction. DB is asymmetrical while COIF is almost symmetrical. Scaling and wavelet functions for decompositions and reconstruction in the BIOR family can be similar or dissimilar. Daubechies wavelets are the most popular wavelets and represent the foundation of wavelet signal processing and are used in numerous applications. The wavelets are then selected based on their shape and their ability to compress the image in a particular application. The most promising results for grayscale compression are provided by Biorthogonal wavelet filter. The Biorthogonal wavelets can use filters with similar or dissimilar order for decomposition and reconstruction. Therefore Biorthogonal wavelet is parameterized by two numbers and filter length is {max (2Nd, 2Nr) +2}. Higher filter orders give higher degree of smoothness [20].

5. Methodology of Image Compression

An image was selected & different wavelets with different filter orders; threshold values of 10 to 100 were applied empirically by the variation of 10. Different levels of decomposition from 1 to 10 were applied. Finally, we conclude that the best results are obtained for level 4 of decomposition at threshold 10, in each case of all images & each wavelet family.

6. Results and Discussion

Wavelet family	CR	PSNR
Db	98.88	49.58
Symlet	97.276	56.27
Biorthogonal	98.87	53.46
coiflet	98.88	48.88

Best results of different wavelet family for bird image

The working methodology is tested on standard test images, Quality of the compressed image depends on image content & size and the compressed image degrades as per level of decomposition because at each level of decomposition there is some loss of energy. It is also found that quality of the degrades rapidly with increasing threshold.

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