ISSN 2349-0292
Impact Factor 2.365

Global Journal of Advanced Engineering Technologies and Sciences

EQUILIBRIA AND STABILITY IN A DISCRETE SIQ EPIDEMIC MODEL

George Maria Selvam[*1], P.Shobana[2]
[*1]Sacred Heart College, Tirupattur - 635 601, S. India.
[2]Sacred Heart College, Tirupattur - 635 601, S. India.

Abstract
A discrete-time SIQ epidemic model described by difference equations is proposed and the stability properties are studied. The basic reproductive number of the discrete SIQ epidemic model is computed. The stability the disease free equilibrium and the endemic equilibrium are demonstrated. Numerical simulations are performed to illustrate the theoretical results.

Keywords: Formulation Of The Model Equilibria & Disease Free Equilibrium.

Introduction
Qualitative descriptions of disease dynamics allow us to understand the behavior of infection within an individual and shed some light on potential transmission. One of the primary reasons for studying infectious diseases is to improve control and ultimately to eradicate the infection from the population. Mathematical models have become an important tool in analyzing the causes, dynamics, and spread of epidemics. Continuous-time models using differential equations were proposed to describe epidemics spreading [3]. Discrete-time models have gained substantial importance during the last years since we can find the simplicity of its simulation, the fact that it is easier to adjust systems parameters from statistical data in discrete-time models. These models may exhibit a richer dynamic behavior than its continuous-time counterparts. The discrete epidemic models focused on the computation of the reproduction number, the existence and stability properties of the equilibrium points, and the extinction, permanence, and persistence of the disease.

The technique of introducing quarantine in standard and epidemic models has received great interest in the last two decades. Over the centuries quarantine succeeded to reduce the transmission of human and animal diseases. Also quarantine can lead to periodic solutions.

Formulation Of The Model Equilibria
An epidemic is an outbreak of a disease over a short time period; a disease is said to be endemic if it persists in a population over a long period of time. We include a class Q of quarantined individuals, who have been removed and isolated from the infectious class. For some milder diseases, quarantined people could be people who choose to stay home from school or work because they are sick. For other more severe diseases, quarantined people could be those who are forced into isolation. It is assumed that these quarantined individuals do not mix with others, so that they do not infect susceptibles. S, I, Q denote the number of susceptible, infected, isolated (quarantined) populations [1,5]. In this section, we analyze the following discrete SIQ epidemic model.

$$\begin{aligned} S(n+1) &= A+(1-m)S(n)-\frac{bS(n)I(n)}{A} \\ I(n+1) &= [1-(m+c)]I(n)+\frac{bS(n)I(n)}{A} \\ Q(n+1) &= [1-(m+e)]Q(n)+cI(n) \end{aligned} \qquad (1)$$

Where $A, m, b, c, e > 0$ and the initial conditions are $S(0), I(0), Q(0) > 0$.The parameter have the following meaning: b is the birth rateof infectives caused by the disease. The system(1) always has a disease–free equilibrium $E_0 = \left(\frac{A}{m}, 0, 0\right)$. E_0 is called the disease free equilibrium since I classes are empty.

Dynamic Behavior Of The Model And Numerical Simulations

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The basic reproductive ratio is one of the most critical epidemiological parameters. It defines the average number of secondary cases an average primary case produces in a totally susceptible population. This parameter allows us to determine whether a disease can successfully spread or not. The Jacobian matrix of system (1) is

$$J(S,I,Q)=\begin{pmatrix} (1-m)-\frac{bI}{A} & -\frac{bs}{A} & 0 \\ \frac{bI}{A} & 1-(m+c)+\frac{bS}{A} & 0 \\ 0 & c & 1-(m+e) \end{pmatrix} \quad (2)$$

DISEASE FREE EQUILIBRIUM

At the disease-free equilibrium, the matrix of the linearization is given by

$$J(E_0)=\begin{pmatrix} (1-m) & -\frac{b}{m} & 0 \\ 0 & 1-(m+c)+\frac{b}{m} & 0 \\ 0 & c & 1-(m+e) \end{pmatrix}$$

The eigen values of the matrix $J(E_0)$ are $\lambda_1 = 1-m$ and $\lambda_2 = 1-(m+e)$ and $\lambda_3 = 1-(m+c)+\frac{b}{m}$.The basic reproductive number, R_0 ,is fundamental in the study of epidemiological models. Here the basic reproductive number $R_0 = \frac{b}{m(m+c)} < 1$.The epidemics preads when $R_0 > 1$ and dies out when $R_0 < 1$. If $R_0 < 1$, the disease-free equilibrium E_0 is stable.

Example 1. We choose the parameter values A= 0.4; b=0.5; c=0.45; m=0.55; e=0.037. Here $R_0 = \frac{b}{m(m+c)} = 0.9090 < 1$; So the equilibrium point is globally stable, See fig-1.

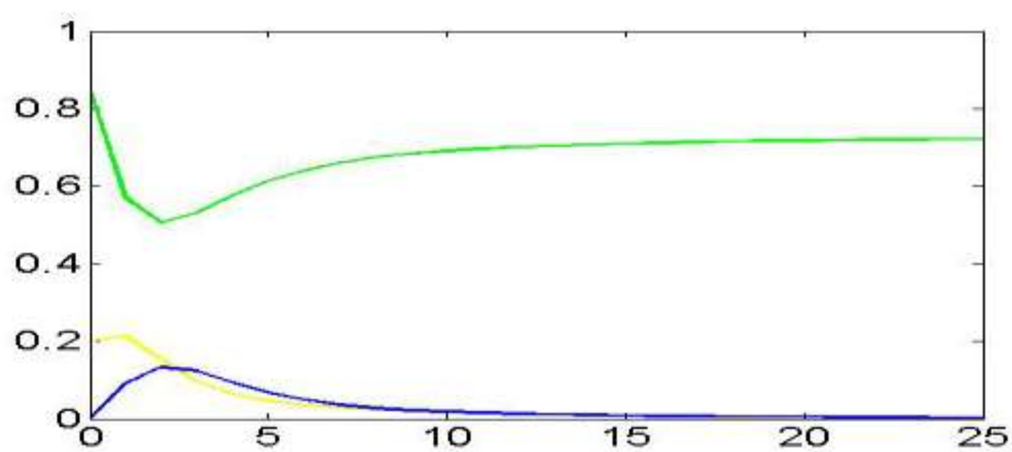

Fig-1 Time plot and phase diagram for the system (1) with R_0<1

Thus the disease free equilibrium of (1) is asymptotically stable when $R_0 < 1$. In the following figure (1), the effect of the parameter on the disease dynamics (infection) is demonstrated.

Example 2. Choose the parameterA=0.4; b=0.1; c=0.45; m=0.055; e=0.037.Here $R_0 = \frac{b}{m(m+c)} = 39.916 > 1$;

So the equilibrium point E_0 is asymptotically stable, see fig -2

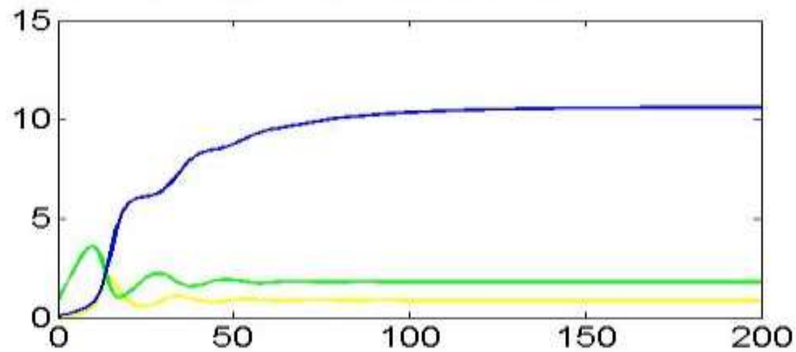

Fig-2 Time plot and phase diagram for the system (1) with R_0>1

When $R_0 > 1$,the average number of a new infection by an infected individual is more than one. Hence the disease may keep persistent in the population. The discrete SIQ model considered in this paper is simple, but it exhibits rich and complicated dynamical behavior. The analytical findings are confirmed with numerical simulations.

VARIATION OF R_0

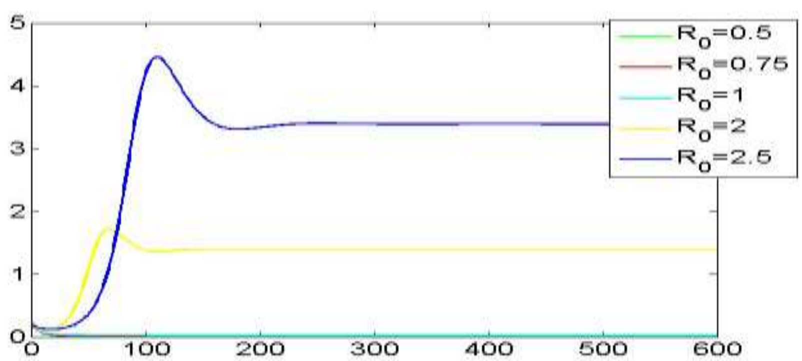

Fig-3 variation of R_0

VARIATION IN I

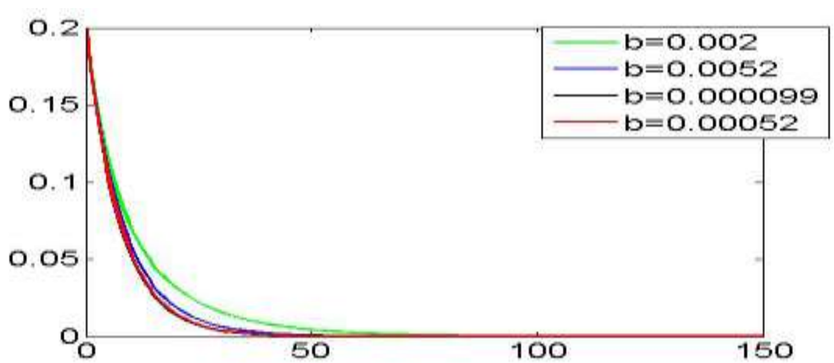

Fig-4 variation of I

Theorem 1. The stability of E_0 is if $|\lambda_{1,2}| < 1$ is a sink and then $|\lambda_{1,2}| > 1$ is a source.

Proof:

The jacobian matrix at the disease free equilibrium $E_0 = \left(\frac{A}{m}, 0, 0\right)$ is

$$J(E_0) = \begin{pmatrix} (1-m) & -\frac{b}{m} & 0 \\ 0 & 1-(m+c)+\frac{b}{m} & 0 \\ 0 & c & 1-(m+e) \end{pmatrix}$$

$$\lambda_2 < 1 \Rightarrow m < 2-e \text{ and } \lambda_3 < 1 \Rightarrow m < (2-c)+\frac{b}{m}$$

Here $|\lambda_{2,3}| < 1$ are a sink.

$$\lambda_2 > 1 \Rightarrow m > 2-e \text{ and } \lambda_3 > 1 \Rightarrow m > (2-c)+\frac{b}{m}$$

Here $|\lambda_{2,3}| > 1$ are sources.

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