

GLOBAL JOURNAL OF ADVANCED ENGINEERING TECHNOLOGIES AND SCIENCES**ANALYSIS OF PERIPHERAL PRESSURE VARIATION FOR FIVE INLINE CYLINDERS IN UNSTEADY LAMINAR CROSS FLOW****Gautam Kumar Mandal*, Dr. T. Suthakar***PGP, IIM Ahmedabad, Mechanical Engineering, NIT Trichy, India
Professor & HOD, Mechanical Engineering, NIT Trichy, India**ABSTRACT**

The effect of pressure variation and fluctuating forces along the periphery of five inline cylinders has been studied numerically at Reynolds number 200. The numerical methodology and the code employed to solve the Navier–Stokes and continuity equations in an unstructured finite volume grid are validated for the case of flow past two tandem cylinders at three spacings. Further, adopting the validated methodology, flow over five inline cylinders is numerically investigated for nine different gap ratios. The Reynolds number is based on the diameter of cylinders, D . The wide range of gap ratio chosen in this paper provide significant insight into the variations of drag and lift forces with time as well as the variation of pressure coefficient which not only varies with time but with the angle along the cylinder periphery also. The flow behavior was observed with increasing gaps. The post processing work was done for data which was not directly available from ANSYS Fluent and fluctuation of force coefficients as well as pressure coefficient was analyzed for each cylinder yielding meaningful graphs. The results may have direct practical application for safe and wise design of cylinder like components in fluid flow.

KEYWORDS: Inline Cylinders, Hydrodynamic Forces, Cylinder Array, Gap Ratio, Pressure Coefficient.**INTRODUCTION**

Flow past a circular cylinder has been a major research area in fluid mechanics in recent years because of the great practical importance in engineering, applied in cases of heat exchangers tube arrays, offshore structures, overhead cables, steam-boiler tube arrays and micro electro mechanical systems as well. At a very low Reynolds number ($Re = UD/\nu \ll 1$), flow around a circular cylinder is steady and symmetrical upstream and downstream, where U is the free-stream velocity, D the cylinder diameter and ν the fluid kinematic viscosity. As the Reynolds number increases, the upstream-downstream symmetry disappears behind a cylinder. These eddies become bigger with increasing Reynolds number. For Reynolds number 50 or above, the flow shows unstable behavior. This is because the critical Reynolds number is approximately $Re = 47$, as predicted by the linear theory of stability.

Also, when vortex shedding occurs behind a circular cylinder, drag on the cylinder increases and body suffers from a periodic forcing in the normal direction to the main stream. These forces may shorten the life of the structure. The complexity of flow separation and free shear layer interference generated by the cylinder arrays has attracted considerable attention in the past. In many of the engineering applications, Karman vortex shedding is responsible for problems with flow-induced vibration and noise. A complete understanding of the fluid dynamics for the flow past inline cylinders includes fundamental subjects as the boundary layer separation, the free shear layer, the wake, and the dynamics of vortices. When two or more circular cylinders are placed in close proximity to one another, the study becomes relatively difficult as well more close to realistic situations. The study in this regard started with cases of two inline cylinders over a decade ago when, the wake interaction between two was studied experimentally by some researchers such as **Bearman and Wadcock**[15], **Zhang and Melbourne** [26] and **Ryu et al.** [27]. Flow over two cylinders using a stabilized finite element method and their study for the Reynolds numbers of 100 and 1000 in tandem and staggered arrangements for different spacing was reported in 1997, by **Mittal et al.** [13]. **Meneghini et al.** (2001) [28] employed a 2-D finite element method to study the vortex shedding and wake interference between tandem and side-by-side cylinders at $Re = 200$ at different spacing. **Chunlei Liang et. al.** 2009 [32] has analyzed the effect of gap ratios at $Re=100$ without much focus on pressure variation. **Ding et al.** (2007)[14] has employed a mesh-free least square-based finite difference method to investigate the flow past two tandem and side-by-side cylinders for the Reynolds numbers of 100 and 200. However, the picture becomes clear only when more number of cylinders is present.

‘A gallery of fluid motion’ by **Cambridge University press (2003)** [29] shows some standard flow behaviors which were further treated rigorously in the works of **K.Lam and K.T.Chan 2003** [30] and **K.Lam and W.Q.Gong (2008)**[31]. Among other research done in recent years, one is by **Singha and Sinhamahapatra**, (2010) [17] who employed a second-order implicit finite volume method to study the laminar flow around two circular cylinders at $40 \leq Re \leq 150$. This is an elementary but important work in this direction. Experimental investigation of flow through a bank of cylinders of varying geometry has been done in 2009 by **S. Gilchrist and S. Green** [33]. Numerous researches have been focused at limited number of gap ratios were flow behavior is

studied in great detail. In this paper we have tried to focus more on the pressure variation and fluctuation of forces over the cylinders.

NUMERICAL METHODOLOGY AND GOVERNING EQUATIONS

The finite volume method has been used in this study to solve the governing equations of fluid flow. The basic principles of conservation of mass, momentum and energy when applied to fluid flow lead to the governing equations of fluid flow. They are popularly called as the Navier-Stokes equations. Solving any problem in fluid mechanics numerically basically means solving these equations under some conditions specific to the problem. We have used the commercial software **ANSYS Fluent** for the solution of these equations. Conservation of mass applied to the fluid flow leads to the continuity equation which can be written as:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho u) = 0 \quad (1)$$

This equation is frequently written in many other forms using different notations and pertaining to different situations. Similarly, the momentum conservation equation in its most useful form, as far as present study is concerned, can be written as:

$$\rho \frac{\partial(u)}{\partial t} = -\frac{\partial p}{\partial x} + \text{div}(\mu * \text{gradu}) + S_{Nx} \quad (2)$$

And

$$\rho \frac{\partial(v)}{\partial t} = -\frac{\partial p}{\partial y} + \text{div}(\mu * \text{gradv}) + S_{Ny} \quad (3)$$

In the above equations, u and μ refer to velocity and dynamic viscosity of the fluid respectively. Symbol ρ denotes the density of fluid which is treated constant here as we are dealing with incompressible fluid flow. Without considering the body forces in much detail their overall effect can be included by defining a source S_{Mx} of x-momentum per unit volume per unit time. Similarly, we have S_{My} for y-momentum. The z-momentum equation is not required as we are dealing with 2-dimensional flows. For detailed discussion on governing equations of fluid flow, one may go through reference [9].

FLOW PAST TWO INLINE CYLINDERS

This section includes the validation methodology to be adopted for the case of five inline cylinders by comparing the results obtained in the case of two inline cylinders with established experimental and numerical data. As our main focus is flow past a cylinder array, we shall briefly explain this validation work and present a comparison table.

Computational domain and Meshing: Computational domain for five inline cylinders is shown in Fig.1. In the case of flow over just two inline cylinders, the other three cylinders will be absent. Other dimensions remain the same as shown. The cylinder is simulated with a diameter (D) of 1 cm. An unstructured triangular mesh is employed in these simulations. Unstructured mesh is generated using ANSYS GAMBIT and then imported into ANSYS Fluent. The grid near the cylinder surface of the domain was kept denser and fine because in that location very fine mesh is required to resolve boundary layer separation and the vortex street.

Boundary conditions: Longitudinal uniform velocity of 2m/s is introduced at the inlet correspond to the Reynolds number 200. The outlet boundary is defined with an average static reference pressure of 0 Pa. The cylinder wall is no slip wall boundary condition where velocity increases from zero at the wall surface to the free stream velocity away from the surface and the rest of the boundaries (top and bottom wall) are symmetry boundary condition.

Result and Conclusion: First mesh independency test was performed through different grids at $Re = 100$. We took four types of grids- **coarse, standard, fine and very finer** and it was revealed that the standard mesh is best for our study. Grids fineness in the region of cylinder was controlled by number of nodal points on cylinder circumference. The non-dimensional time step Δt was set to approximately 0.02, to maintain the Courant number less than 1 for the stability of solution.

The drag coefficient and lift coefficient as a function of the dimensionless time for different values of the Reynolds number were calculated. These results were obtained using Equation 4 and 5 written below:

$$C_d * \text{density} = F_d / (0.5 U^2 A) \quad (4)$$

$$C_l * \text{density} = F_l / (0.5 U^2 A) \quad (5)$$

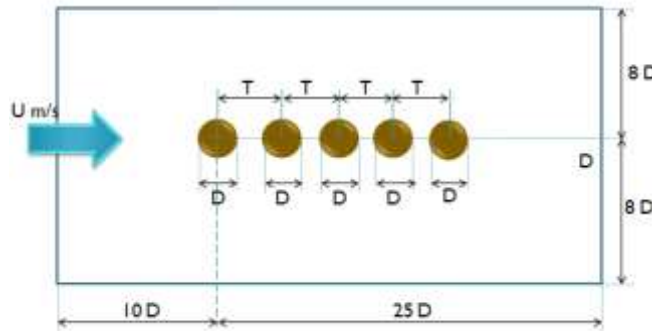


Fig. 1. Schematic of Computational domain for five inline cylinders

where A is the projected area in the flow direction and F_d is the sum of the pressure force and the viscous force components on the cylinder surface acting in the horizontal direction whereas F_1 is that in the vertical direction. The study of pressure variation in such cases is aided by the non-dimensional parameter pressure coefficient, defined as

$$C_p \cdot \text{density} = (P_0 - P) / (0.5 U^2) \tag{6}$$

where, P is oncoming flow static pressure and P_0 is the static pressure at any point on the surface of cylinder. In table 1, we have presented the comparison between the obtained values of mean drag coefficient C_d and Strouhal number S ($S = fD/U$, f being vortex shedding frequency) in the present work from the established numerical and experimental data. This table corresponds to two inline cylinders. UC and DC denotes upstream and downstream cylinders. As the results obtained are in very close agreement with the established literatures, this suggests that a similar numerical approach shall be desirable for study of flow over five inline cylinders as well and would yield meaningful results.

Table 1: Comparison of flow parameters for two inline cylinders at $Re = 200$

Parameter s	L/D	Mean drag coefficient C_d					Strouhal number S				
		Present	Hesam et al. [10]	Meneghini et al. [14]	Mahir and Atlac [20]	Slauti and Stansby [26]	Present	Hesam et al. [10]	Meneghini et al. [14]	Mahir and Atlac [20]	Slauti and Stansby [26]
1.5	UC	1.41	1.05	1.06	-	-	0.166	0.175	0.167	-	-
	DC	-0.21	-0.15	-0.18	-	-	0.166	0.175	0.167	-	-
2	UC	1.083	1.03	1.03	1.06	0.89	0.145	0.138	0.13	-	0.13
	DC	-0.23	-0.16	-0.17	-0.21	-0.21	0.145	0.138	0.13	-	0.13
4	UC	1.336	1.16	1.18	1.34	1.11	0.181	0.179	0.174	0.181	0.19
	DC	0.709	0.52	0.38	0.558	0.88	0.181	0.179	0.174	0.181	0.19

The excellent agreement for two inline cylinders shows that our choice of computational domain, standard mesh and working method is correct. Thus, we now proceed to flow over five inline cylinders using the same methodology.

FLOW PAST FIVE INLINE CYLINDERS

In this section, numerical simulation of the 2-D unsteady laminar flows around five inline cylinders has been discussed. The computations were carried out at $Re = 200$ for the spacing of $L/D = 1.2, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5$ and 5.0 . It is remarkable that here 9 gap ratios are used. This has widened the scope of study hence more practical situations are covered. For example, nitrogen gas cylindrical pipelines in steel industry are generally kept at spacing $L/D > 4$, say 5, while offshore structure may have the cylinder like structure very close in many cases, say $L/D = 1.2$. The changes happening in the flow behavior with changing gap ratios were studied with careful examination of pressure field, vorticity field and streamlines for each gap ratios. The phenomenon is that of flip-flopping has been best observed in our work at $L/D = 2.5$. Numerous research works has already been carried out in studying these patterns for two cylinders e.g. Lui et al [18] at limited gap ratios. Here, we present **the vorticity plots in nine continuous gap ratios** which is unique to this research work.

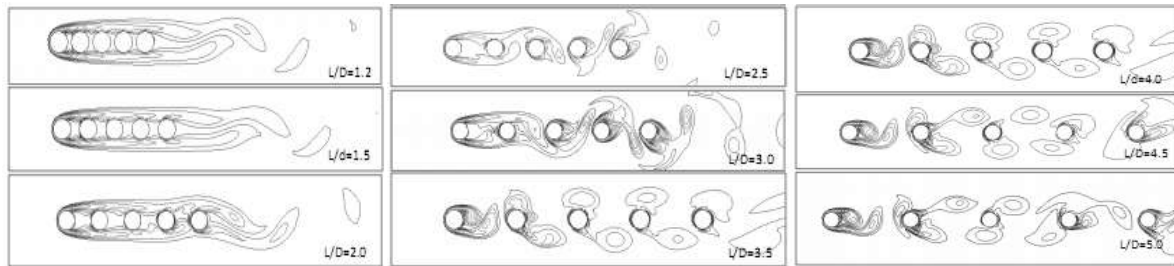


Fig. 2. Vorticity at nine gap ratios ($L/D = 1.2, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0$)

RESULT AND DISCUSSION

It was seen that as the gap ratio was increased, the vortex shedding was more intense due to stronger eddies formed. The reason of smooth eddies at close gaps can be easily understood from the fact that T/D of 1.2 will mean that the closest points on two cylinders are as close as one-fifth of their diameters. The fluid doesn't enter this narrow region and easily flows over the cylinders showing longer patterns at lower spacing. Once this longer pattern passes all the cylinders, there is no interference and the eddies die soon. For higher ratios, there are large eddies as the fluid gets more gap to go inside and vortices from two sides intermingle with each other being close to each other. Notice that the eddies have become very intense at gap ratio above 3.5.

From the observations made in ANSYS Fluent, at $L/D = 1.2$ and 1.5, we noticed that cylinder 2 and 3 experience negative drag. This is because low pressure zones form between cylinder 1 and 2, and cylinder 2 and 3. See Fig. 3 for dynamic pressure contours. Due to similar reasons, at $L/D = 2.0, 2.5, 3.0$; only cylinder 2 is showing negative drag due to low pressure zone between cylinder 1 and 2. At higher gaps, the pressure contour approaches uniformity hence pressure variation is less. Also notice that for higher gap ratios, leaving the first cylinder; all other cylinders have very less pressure difference between its upstream and downstream. This concept can have direct practical use in design of cylindrical array like structure where we need to make the first cylinder stronger than others because this one faces higher pressure difference. The stronger eddies at higher gaps as shown in vorticity contours are also supported by pressure distribution.

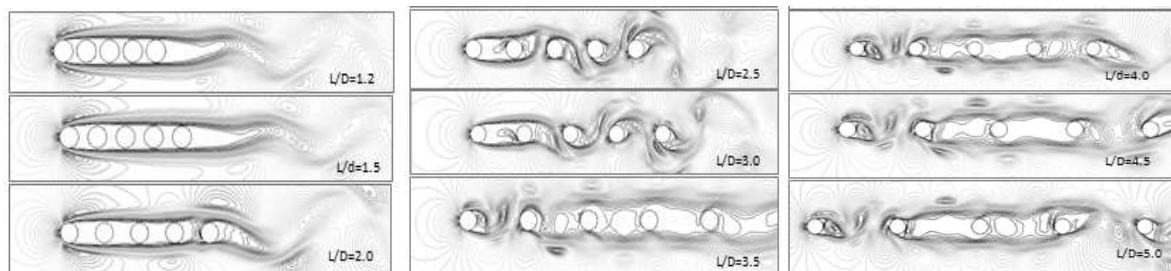


Fig. 3. Dynamic pressure contour at nine gap ratios ($L/D = 1.2, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0$)

The same idea becomes clearer when we try to plot the pressure coefficient for specific gap ratios which is completely new perception for analyzing the behavior of cylinders at varying gaps, adopted in this research work. Earlier, some authors like Chunlie *et al* [32] tried to analyze cylinder arrays by using hydrodynamic force coefficient at a lower Reynolds number of 100 as compared to 200 here. In this work, we shall observe both pressure coefficients and force coefficients for different gaps at higher Reynolds number of 200. The choice of higher Re was inspired by the flow related to offshore structures and in backwater underwater cylindrical columns where Reynolds number is not very high but essentially greater than 100. Fig. 4. Shows C_p values for all five cylinders, which clearly represent the magnitude of dynamic pressure corresponding to each cylinder in comparison to others, clearer than what one would observe from looking at just the pressure contours.

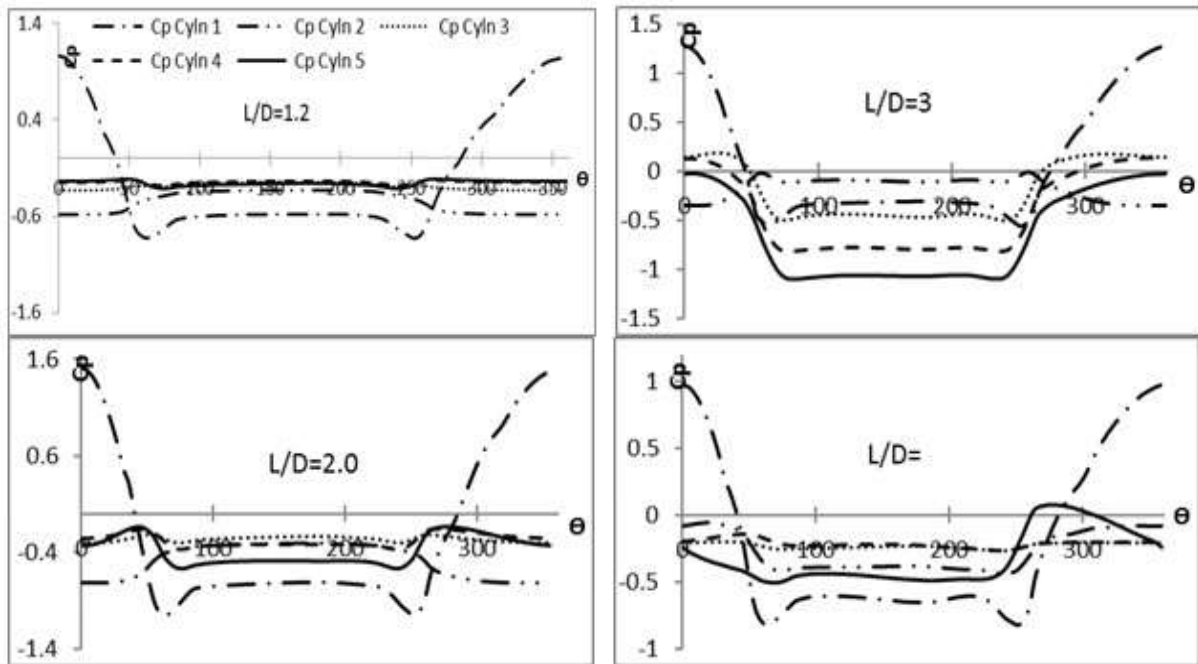


Fig. 4. Pressure coefficients at increasing gaps

In this research work, we discovered that $L/D=3$ is the critical gap ratio for inline cylinders. We have selectively shown C_p variation at four gaps including $L/D = 3$, from which it appears that the trend is different for the gap ratio of 3. While between the two separation points, the pressure varies for almost all cylinders for different gap ratios, not so is the case when gap ratio is 3. Here we notice that the pressure remains fairly constant between the separation points. Also, the behavior of cylinder 5 is amazingly different for $L/D = 3$ as compared to that at other gaps. In practice, often it is not C_p but its variation which is more detrimental for a system. The behavior of system closely depends of the fluctuations in pressure at different gaps and the choice of proper gaps may accordingly be decided. Fig. 5 shows C_p' for four gap ratios. At first sight, the fluctuations seem to be totally random for the cylinders. However, a close examination will show the systematic increase in fluctuations with increasing gaps for each cylinder. The fluctuations in cylinder 5, the last in the array is prominent. This actually demonstrates the need to pay special attention to the last cylinder when numerous inline cylinders are placed.

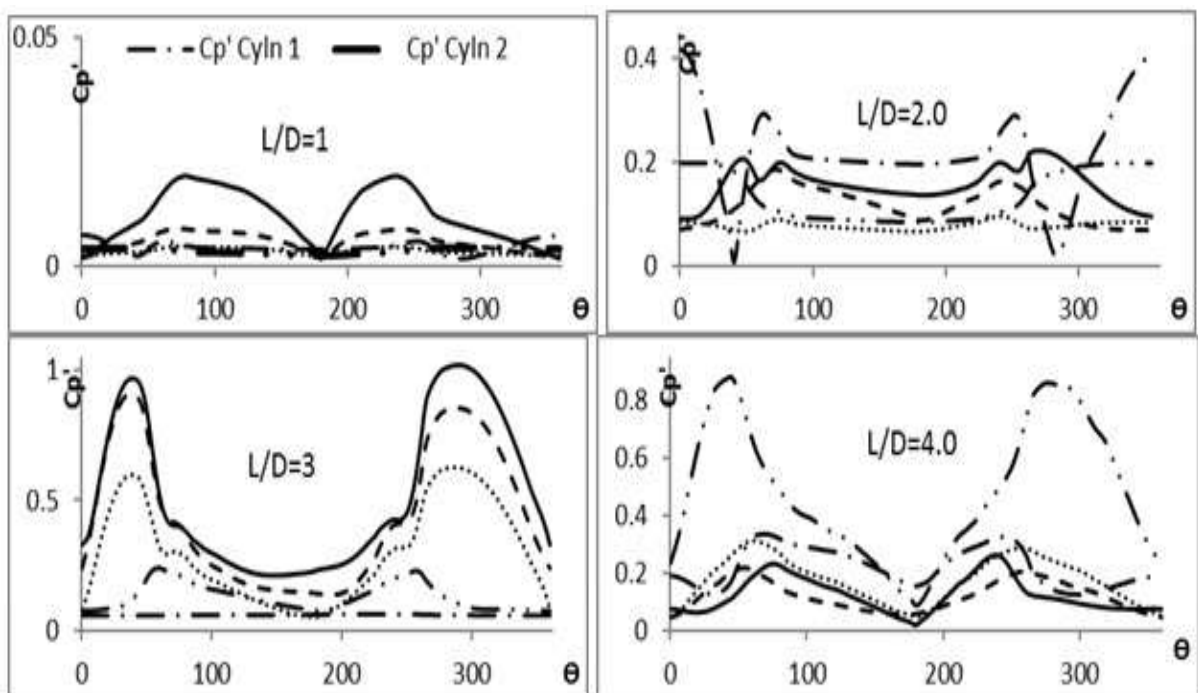


Fig. 5 C_p' fluctuation of pressure coefficients at increasing gaps

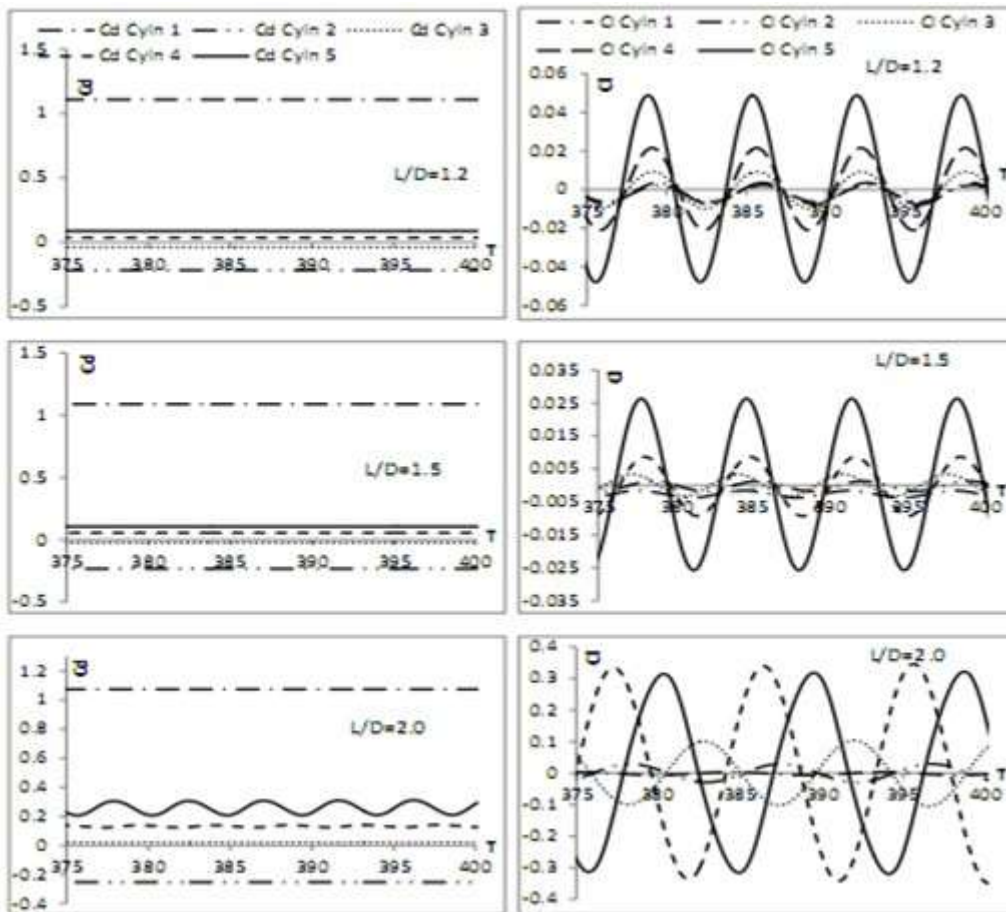


Fig. 6(a). Drag and Lift behavior at lower gaps

The fluctuations in pressure coefficients is an innovation, and we can see that in case of ratio $L/D = 3.0$ the last cylinder shows high fluctuation, but noticing the C_p graph we observe that it is negative for the last cylinder for almost whole periphery. This means pressure had suddenly reduced. In any engineering application, thus, the last of the inline cylinders must be designed keeping in mind the fact that the last cylinder is going to experience sudden fluctuations. The drag and lift forces also show a remarkable change in behavior as cylinders are placed far away from each other. The behavior at lower gaps is first shown. Drag the cylinders remain almost constant while lift shows uniform periodic variation with time.

As gaps increase, drag is no more constant and starts fluctuating for all the cylinders. This observation can also be supported by vorticity plots. The amplitude of lift fluctuations are generally higher at higher gaps as shown in Fig.6(b).

Additionally, we also introduce the mean drag and lift fluctuations for five cylinders in this study. This shall present an easy to observe picture of how the hydrodynamic forces fluctuate more or less with varying gap spacing. Recall that we observed that at very low spacing the eddies are smooth and die out soon. This can be understood by the constant C_d values at low spacing. However, as the spacing increase, the drag coefficient may take lower or higher value depending on the cylinder position and the C_d now obeys fluctuating nature. The transition region is the intermediate gap ratios.

From the perception of hydrodynamic histories, the behavior of mean value was studied for each cylinder with respect to gap ratios. This shows interesting nature as shown in Fig. 7. Critical gap of $L/D = 3$ shows an increase in mean C_d for all cylinders which is a remarkable phenomenon.

In a similar fashion, the variation of lift coefficient has been presented with varying L/D . Generally, the lift forces are always present whatever be the gap ratios and this may lead to vortex induced vibrations as well. Optimum spacing may be chosen for permissible fluctuations in lift. Based on the drag and lift coefficient, a phase diagram may be plotted between them which will demonstrate the possible locus of centroid of cylinders (in a proportion),

if cylinders are allowed to vibrate. Needless to mention, this locus shall vary for various gap ratios. This means that without actually performing simulation for vibrations, we may anticipate them in possible cases.

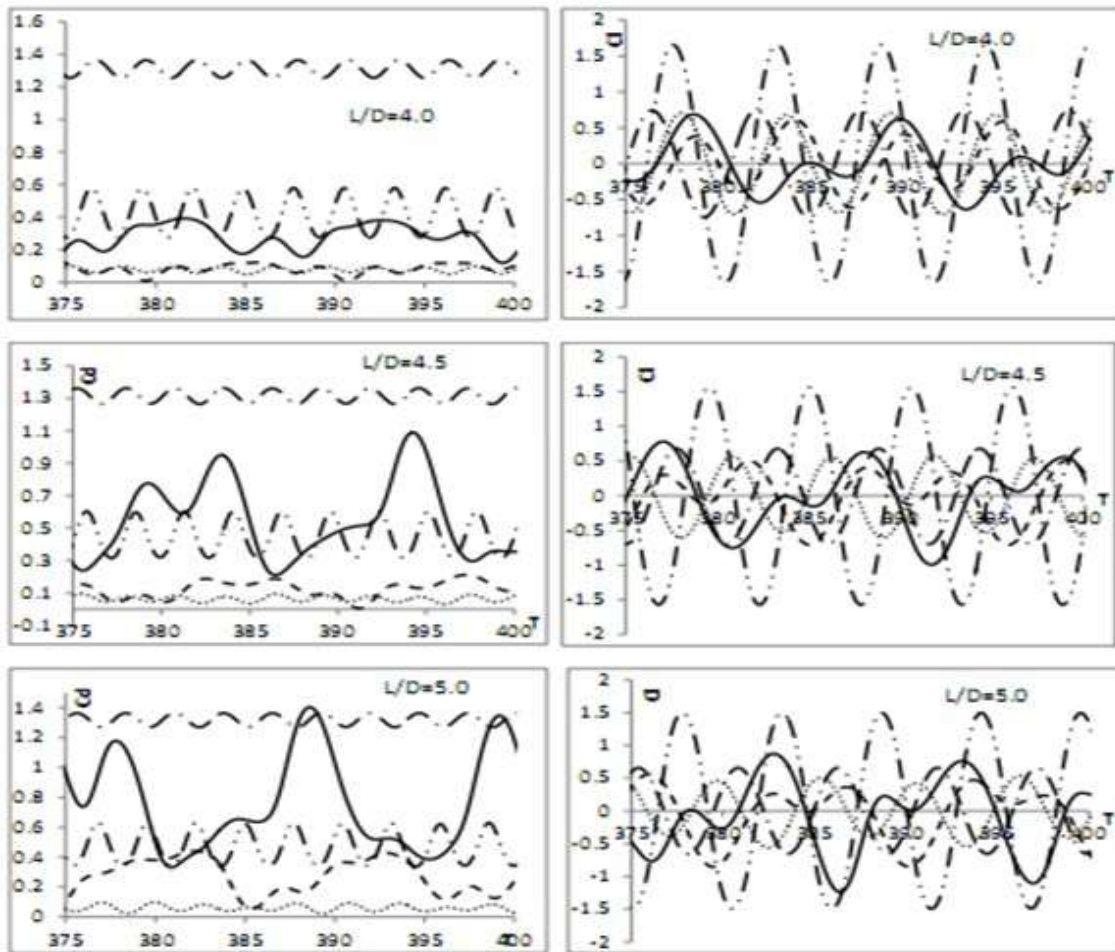


Fig. 6(b). Drag and Lift behavior at higher gaps

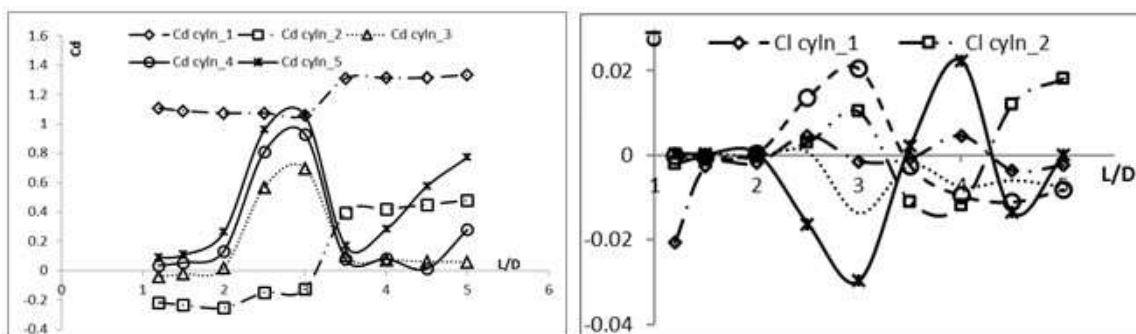


Fig. 7. C_d , C_l , C_d' and C_l' with changing L/D

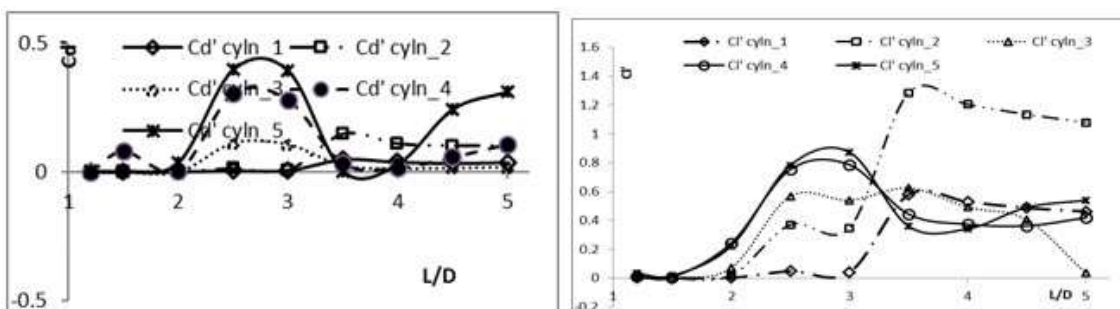


Fig. 7 (b). C_d , C_l , C_d' and C_l' with changing L/D

CONCLUSION

In this research work, the variation of pressure is studied which varies not only with time but with respect to the angle chosen in the periphery of the cylinders. We have time averaged the C_p , coefficient of pressure and taken its value at different points in the cylinder periphery. The fluctuations of C_p hints that in case of designs where many inline cylinders are placed, the last one should be designed keeping in view the relatively larger fluctuations to be experienced by that cylinder. The study of pressure variation shows the point of maximum pressure for the five cylinders. In any practical situation, chances of failure are more likely to start from these points. Accordingly, the design may use the choice of increasing or decreasing the gap which causes the maximum pressure points to get inwards or outwards hence the supports for the cylinders need to be stronger in that particular direction only. Also, for $L/D=3$, the value between separation points are almost same. Designing any component near the critical spacing needs extra care. At lower gaps, the separation points are close for the two cylinders which shall be desirable in most of the practical applications because this reduces overall eddies. Insufficient information about the pressure variation may lead to poor designs. For example, notice the trend of C_p , the minima of fluctuations for most of the spacings occurs at angle near to 180° whereas near that angle for $L/D=2.0$, fluctuations are high near 180° . This means even if we are designing similar arrays but with different gaps between tubes, proper care has to be taken. Lastly, it is worth mentioning that the huge post processing methods incorporated in this research shows a wider future work scope.

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