Global Journal of Advance Engineering Technologies and Sciences

APPLICATION OF GRAPH COLOURING IN COMPUTER SCIENCES

Berdewad O. K. & S.D.Deo
Department of Mathematics, NES College, Bhadrawati,Dist.Chandrapur, India.
GondwanaUniversity ,Gadchiroli, MS, India.

ABSTRACT
The field of mathematics plays significant role in numerous fields. One of the essential areas in mathematics is graph theory which is used in structural models. This structural arrangements of several objects or technologies lead to new developments and modifications in the existing environment for enhancement in those fields. The field graph theory started its journey from the problem of KoinsbergBridge in 1735. This paper has been gives the historical background of graph theory and important application of graph colouring such as time table scheduling ,job scheduling, map colouring and GSM mobile phone network.

Keyword:- Graph Colouring, Map Colouring, GPS.

HISTORY OF GRAPH THEORY

The origin of graph theory in progress with the problem of Koinsberbridge, in 1735.This problem lead to the idea of Eulerian Graph. Euler studied the problem of Koinsbergbridge and built a structure to solve the problem named Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles[2]) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem.Then in 1856, Thomas. P. Kirkman and William R.Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H.Dudeney mentioned a puzzle problem. Eventhough the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory.

Graph coloring is a fundamental combinatorial optimization problem arising in several applications in computer sciences. The problem has been traditionally motivated by applications in time-tabling, scheduling, resource allocation, etc.[1]but also arises in diverse area such as information theory and statistical physics. The problems in this paper are motivated by applications in resource allocation and scheduling. Graph coloring is one of the most important concepts in graph theory and is used in many real time applications in computer science [4]. Various coloring methods are available and can be used on requirement basis. The proper coloring of a graph is the coloring of the vertices and edges with minimal number of colors such that no two vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph [2].

GRAPH THEORY APPLICATION

This section explores the applications of graph especially in computer science in brief. Various applications that deal with computers are using graph theory concepts. Some applications are discussed here

TIME TABLE SCHEDULING

Allocation of classes and subjects to the professors is one of the major issues if the constraints are complex. Graph theory plays an important role in this problem. For m professors with n subjects the available number of p periods timetable has to be prepared. This is done as follows.

A bipartite graph (or bigraph is a graph whose vertices can be divided into two disjoint sets *U* and *V* such that each edge connects a vertex in *U* to one in *V*; i.e., *U* and *V* are independent sets[2]) G where the vertices arethe number of professors say m_1, m_2, m_3, m_4, ...m_k and n number of subjects say n_1, n_2, n_3, n_4, ... n_m such that the vertices are connected by p_i edges. It is presumed that at any one period each professor can teach at most one subject and that each subject can be taught by maximum one professor. Consider the first period. The timetable for this single period corresponds to a matching in the graph and conversely, each matching corresponds to a possible assignment of professors to subjects taught during that period. So, the solution for the timetabling problem will be obtained by partitioning the edges of graph G into minimum number of matching. Also the edges have to be colored with minimum number of colors. This problem can also be solved by vertex coloring algorithm.“ The line graph L(G) of G has equal number of vertices and edges of G and two vertices in L(G) are connected by an edge iff the corresponding edges of G have a vertex in common. The line graph L(G) is a simple graph and a proper vertex coloring of L(G) gives a proper edge coloring of G by the same number of colors. So, the problem can be solved by

finding minimum proper vertex coloring of L(G) ." For example, Consider there are 4 professors namely m_1, m_2, m_3, m_4,.and 5 subjects say n_1, n_2, n_3, n_4, n_5 to be taught. The teaching requirement matrix p = [p_{ij}] is given below.

P	n_1	n_2	n_3	n_4	n_5
m_1	2	0	1	1	0
m_2	0	1	0	1	0
m_3	0	1	1	1	0
m4	0	0	0	1	1

Figure: The teaching requirement matrix for four professors and five subjects.

The bipartite graph is constructed as follows.

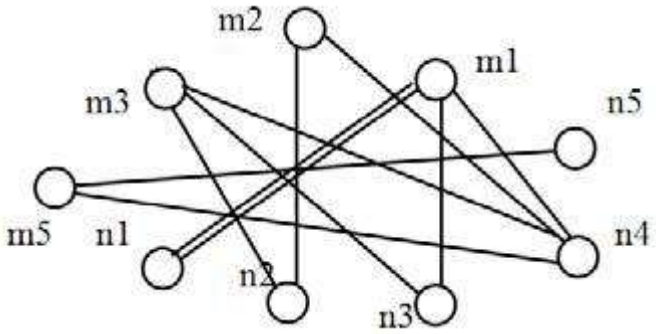

Figure: Bipartite graph with 4 professors and 5 subjects.

Finally, the authors found that proper coloring of the above mentioned graph can be done by 4 colors using the vertex coloring algorithm which leads to the edge coloring of the bipartite multigraphG.Four colors are interpreted to four periods.

----	1	2	3	4
m1	n1	n1	n3	n4

SCHEDULING CONFLICTING JOBS

In several scheduling applications, jobs that need to be scheduled require some non-sharable resource to run. For example, processes in a distributed operating sys-tem that need access to the same file or device cannot be scheduled simultaneously. As described earlier, such a scheduling problem can naturally be modeled as a graph coloring problem. However, not all jobs or processes are the same and each process may have different running times. This gives rise to more general coloring problems where each vertex has weight corresponding to its running time and we are interested in minimizing the average completion time of the jobs or the time by which the last job completes, etc. Further options such as whether jobs can be pre-empted, whether jobs have to be executed in batches, etc. give rise to other variants.

MAP COLORING AND GSM MOBILE PHONE NETWORK

Groups Special Mobile (GSM) is a mobile phone network where the geographical area of this network is divided into hexagonal regions or cells. Each cell has a communication tower which connects with mobile phones within the cell. All mobile phones connect to the GSM network by searching for cells in the neighbors. Since GSM operate only in four different frequency ranges, it is clear by the concept of graph theory that only four colors can be used to color the cellular regions. These four different colors are used for proper coloring of the regions. So, the vertex coloring algorithm may be used to assign at most four different frequencies for any GSM mobile phone network.

The authors have given the concept as follows:
Given a map drawn on the plane or on the surface of a sphere, the four color theorem assets that it is always possible

ISSN 2349-0292
Impact Factor (PIF): 2.365

to color the regions of a map properly using at most four distinct colors such that no two adjacent regions are assigned the same color[3]. Now, a dual graph is constructed by putting a vertex inside each region of the map and connect two distinct vertices by an edge iff their respective regions share a whole segment of their boundaries in common. Then proper coloring of the dual graph gives proper coloring of the original map. Since, coloring the regions of a planar graph G is equivalent to coloring the vertices of its dual graph and vice versa.[2] By coloring the map regions using four color theorem, the four frequencies can be assigned to the regions accordingly [3].

Figure: The map of India.

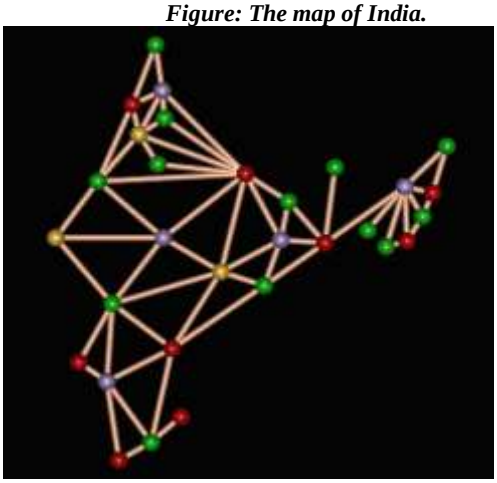

Figure: The dual graph of the map of India

CONCLUSION

An overview is presented especially to project the idea of graph colouring. This paper gives an overview of the applications of graph theory in heterogeneous fields to some extent but mainly focuses on the computer science applications that uses graph theoretical concepts.

REFERENCE

1.Daniel Marx, "Graph Coloring problems and their applications in scheduling.
2. NarasinghDeo, "Graph theory with applications to engineering and computer science", Prentice Hall of India, 1990.
3. ShariefuddinPirzada and AshayDharwadker, "Journal of the Korean Society for Industrial and applied Mathematics, Volume 11, No.4, 2007.
4. S.G. Shrinivaset. al. / International Journal of Engineering Science and TechnologyVol. 2(9), 2010, 4610-4621