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[Cheeneebash *et al.*, 7(10): October, 2021] ISSN 2349-0292
Impact Factor 3.802

GLOBAL JOURNAL OF ADVANCED ENGINEERING TECHNOLOGIES AND SCIENCES

A MULTI OBJECTIVE APPROACH TO THE CHANNEL ASSIGMENT PROBLEM IN A REDUCED SPACE

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ABSTRACT

Wireless communication and mobile computing are evolving rapidly and solving the channel assignment problem has now become a new challenge in research in terms of reduction of the computational time. In other words, there is need in developing a fast mechanism for allocation of channels. In this paper, we present an efficient technique for solving the Channel Assignment Problem (CAP) by using a multi-colouring algorithm in reducing the search space and applying an appropriate tabu search algorithm. We first map a given CAP,*P*, to a smaller subset *P'* of cells of the network, which actually reduces the search space by using a multi-colouring method. Then a tabu search algorithm is applied to solve the new problem *P'*. This method reduces the computing time drastically. To solve the original problem *P*, a modified forced assignment with rearrangement (FAR) operation is applied to *P'*. While solving the CAP, we obtain blocked calls and redundant frequencies. So a multi-objective algorithm, NSGA II is used to minimise the blocked calls and maximise the redundant frequencies. The proposed method has been tested on the well-known Philadelphia benchmark problems. Optimal solutions have been obtained in terms of Pareto fronts, which represent a sort of trade off between blocked calls and redundant frequencies. In case of an unexpected rise in demand, the network manager can use the Pareto front for better decision making

KEYWORDS: Channel Assignment Problem, Reduced Space, Tabu Search, Multi-Objective.

INTRODUCTION

Mobile communication is evolving rapidly with the progress of both wireless communications and mobile computing. However, the frequency bandwidth is limited and an efficient use of channel frequencies is becoming more and more important. For practical purposes, the bandwidth spectrum is divided into a smaller number of channels depending on the service requirement. To satisfy the large demand of mobile users, channels have to be assigned and re-used while avoiding interference and thus increase the traffic carrying capacity of the system. A channel can be used by multiple base stations if the minimum distance at which two signals of the same frequency do not interfere. This is known as the Channel Assignment problem (CAP). In other words, the CAP is considered to be the generalised graph colouring problem, which is a well-known NP-complete problem [15]. In this paper, we assume a cellular network system where the demands of the cells are known a priori, and the channels are to be allocated to the cells statistically to cater sessions that are basically connection oriented. Here, the major aim is to reuse channels in cells while avoiding interference. A channel can be used by multiple base stations if the minimum distance at which two signals of the same frequency do not interfere. A lot of research have been devoted to CAP in the past decades and many methods have been proposed, namely graph theory based methods [28, 3, 15, 12], simulated annealing [9], tabu search [5, 4], neural networks [17] and genetic algorithms [2, 13, 21]. This earlier work can be broadly classified into two categories. The first category algorithm determines an ordered list of calls and then assigns channels deterministically to the calls to minimise the required bandwidth [23, 16, 1, 6, 10]. The algorithms for the second category formulate a cost function, such as the number of blocked calls by a given channel assignment, and try to minimise it with a given bandwidth of the system [9, 17, 20, 24, 19]. In the first category of algorithms, the derived channel assignment always fulfills all the interference constraints for a given demand. On the other hand it may be difficult to find an optimal solution for large and difficult problems. For the second category of algorithms, it may be difficult to minimise the cost function to the desired value of zero for the hard problems, with the minimum number of channels. To evaluate the performance of these algorithms, well known benchmark Philadelphia problems have been solved. These benchmark problems will be discussed in details in sections 2 and 6. The methods mentioned above have been able to solve the CAP with optimal solutions but nowadays the trend that research in this field is mainly towards the need of fast mechanism for allocation of channels within optimal frequency bandwidth. The novelty in this paper is the presentation of the CAP, *P*, in a reduced space*P'* , which is derived by using a multi-colouring method. The authors in [13] have used a completely different technique in grouping the cells. Our technique groups the cells into independent sets. This helps in solving the reduced space more efficiently and in this paper, a tabu search algorithm [14] is used. It has been seen from literature that the ordering of cells is very important in assigning channels and thus the CAP can be seen as a permutation problem. An efficient way for solving such problems is the tabu search algorithm. Once the solution of P' is found, the solution is expanded to the solution of the original problem P. However to solve the original problem P, a FAR algorithm [26] is applied. This method drastically reduces the computation time. In addition, redundant frequencies are found so that these can be used in case there are some changes in the demand vector. Our aim is to minimise the blocked calls given the required bandwidth and on the other hand one can also maximise the use of the redundant frequencies. In [7], a multi-objective approach of the CAP was discussed. The new idea presented in this paper is to solve the CAP from a multi-objective dimension using the reduced space technique. From the reduction of space, we mentioned earlier that redundant frequencies are obtained as well as there are unused frequencies in the bandwidth, so our aim is to maximise these redundant frequencies and minimise blocked calls. It has been observed that in future that the demand for mobile users will keep on increasing as more and more people make use of mobile communications in everyday life. With the new proposed approach, it will be easier to manage the network system in case there are changes in demand in a particular cell. The network manager will have several options for deciding which solution best fits a particular situation. The multi-objective approach gives several possible solutions in terms of blocked calls and redundant frequency in the Pareto set. The paper is structured as follows: section 2 gives the mathematical formulation of the CAP, section 3 presents the tabu search algorithm, section 4 shows how the CAP is reduced using the multi-colouring approach and how we get the final solution using the FAR algorithm. Section 5 describes the multi-objective algorithm and simulation results are discussed in section 6. Finally, we draw some concluding remarks.

MATHEMATICAL FORMULATION OF THE CAP

The CAP in cellular networks is an NP-complete problem [15]. It has been modelled as an optimization problem with binary solutions. The problem is characterized by a number of cells and a number of channels n and f respectively. It must fulfill the following three constraints [12]:

1. The Co-Channel Constraint (CCC): The same channel cannot be assigned to a pair of cells within a specified distance simultaneously.
2. The Adjacent Channel Constraint (ACC): Adjacent Channels cannot be assigned to adjacent cells simultaneously.
3. The Co-site Constraint (CSC): The distance between any pair of channels used in the same cell must be larger than a specified distance

In 1982, Gamst and Rave [12] defined the general form of the CAP in an arbitrary inhomogeneous cellular network. In their definition, the compatibility constraints in an n -cell network are described by an $n \times n$ symmetric matrix called the compatibility matrix $C = [c_{ij}]$. Each non-diagonal element c_{ij} in C represents the minimum separation distance in the frequency domain between a frequency assigned to a cell *i* and one assigned to cell *j*. If $c_{ij} = 0$, it means that a channel assigned to cell i can be reused to cell *j*; $c_{ii} = s$ means the co-site channel interference constraint is s channels. Finally, $c_{ij} = 1$ means that the adjacent channel interference constraint is one channel. The channel requirements are described by an *n*-element vector, which is called the demand vector D. Each element d_i in D represents the number of frequencies to be assigned to cell *i*. The solution of the CAP is represented by a matrix F. Each element of the matrix is defined according to the following expression:

$$a_{ij} = \begin{cases} 1 \text{ if channel } j \text{ is assigned to cell } i \\ 0 \qquad\qquad\qquad\qquad \text{otherwise.} \end{cases}$$

Let the frequencies be represented by positive integers 1, 2, 3, ..., f where f is a maximum allocation of the spectrum bandwidth. The mathematical model can be represented as follows [23, 27]:

1. n: Number of cells in the network.
2. d_i: Number of frequencies required in cell i $(1 \leq i \leq n)$ in order to satisfy channel demand.
3. C: Compatibility matrix, $C = (c_{ij})$ denotes the frequency separation required between cell *i* and cell *j*.
4. f_{ik}: Channel is assigned to k^{th} call in cell *i*.

Therefore the objective of CAP is $\min_{i,k} f_{ik}$, subject to $|f_{ik} - f_{jr}| \geq c_{ij} \forall\, i, j, k \neq r$. The channel assignment problem in the cellular network is to find a conflict free assignment with the minimum number of total frequencies, where *C* and *D* are given, in other words one tries to find the minimum of $\max_{ik} f_{ik}$

Given the above constraints, the CAP can be represented by means of a graph G, where the k^{th} call to cell i is represented as a node v_{ik}, and the nodes and v_{ik} and v_{jr} are connected by an edge with weight c_{ij} if $c_{ij} > 0$. Then, the channels are assigned to the nodes of the CAP graph in a specific order and a node will be assigned the channel corresponding to the smallest integer that will satisfy the frequency separation constraints with all the previously assigned nodes. Thus, one can conclude that the ordering of the nodes is a key factor on the required bandwidth. Let us assume that there exists m nodes in the CAP graph, where m is defined as the total demand, in other words $m = \sum_{i=1}^{n} d_i$. Therefore, the nodes can be ordered in $m!$ ways and finding the optimal ordering is an exhaustive task. In this paper, we have used a tabu search adapted from [14], which is an efficient algorithm for solving permutation problems.

THE TABU SEARCH APPROACH

Tabu search is a heuristic method originally presented by Glover and Laguna [14]. It is a procedure to explore the solution space beyond optimality and exploits the history of the search in order to influence its future steps. A distinguishing feature of tabu search is embodied in its exploitation of adaptive forms of memory, which equips it to penetrate complexities that often confound alternative approaches. We have adapted the tabu search algorithm described in [18], which deals with permutation neighbourhood. We shall now describe the types of insert moves and neighbourhood that has been used in the tabu search. Denoting p to be a random ordering of the n cells. The Insert-Move (p_j , i) function consists of deleting p_j from its current position j to be inserted in position i. Thus one gets the following ordering p' as shown in (1)

$$p' = \begin{cases} (p_1 \dots, p_{i-1}, p_j, p_i, \dots, p_{j-1}, p_{j+1}, \dots, p_n) \text{ for } i < j \\ p_1, \dots, p_{j-1}, p_{j+1}, \dots, p_i, p_j, p_{i+1}, \dots, p_n) \text{ for } i > j \end{cases} \quad (1)$$

Pairwise exchanges or moves are frequently used as one of the ways to define neighbourhoods in permutation problems; identifying moves that lead from one sequence to the next. The neighbourhood N comprises all permutations resulting from executing general insertion moves and is defined as

$$N = \{p': Insert - Move(p_j, i), \text{for } j = 1, \dots, n \text{ and } i = 1, 2, \dots j-1, j+1, \dots, n\}. \quad (2)$$

We define a *first* strategy that scans the list of cells (in the order given by the current permutation) in search for the first cell (p_f) whose movement results in a strictly positive move value. The move selected by the *first* strategy is then Insert-Move(p_f,i^*), where i^* is the position that gives the best move value. The local search is based on choosing the best insertion associated with a given cell. The tabu search procedure starts by generating a random procedure p, and it alternates between an intensification and a diversification phase. The main aim of the search intensification is to explore more thoroughly the portions of the search space that seem "promising" in order to ensure that the best solutions in these areas are indeed found.

Intensification is based on some intermediate term memory, such as a recency memory, in which one records the number of consecutive iterations that various solutions components have been present in the current solution without interruption. The intensification phase starts by a random selection of a cell. The probability of selecting a cell j is proportional to some weight w_j. For our problem, we assign higher weights to those cells having a higher demand. The weight is given as

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j}.$$

The move Insert-Move(p_j; i) ∈ N_j with the largest move value is selected and it becomes the tabu-active for TabuTenure iterations. The number of times that cell j has been chosen to be moved is accumulated in the value *freq(j)*. This frequency information is used for diversification purposes. The intensification phase is terminated after a maximum of a predefined number of iterations is executed without improvement. Before ending this phase, the *first(N)* procedure is applied to the best solution, which is denoted by $\hat{p}$.

Diversification phase is an algorithmic mechanism that forces the search into previously unexplored areas of the search space. It is usually based on some form of long-term memory of a sector and in our case, it is the frequency memory in which one records the total number of iterations that various solution components have been present in the current solution. At each iteration of the diversification phase, a sector is selected randomly and the probability of selecting sector j is inversely proportional to the frequency count *freq(j)*. The chosen sector is placed in the best position, as determined by the move values associated with the insert moves in N_j. This procedure is repeated for a maximum number of iterations.

CONSTRUCTION OF THE REDUCED SPACE

In this section, we describe how the CAP graph *G* is reduced into a smaller space and we use tabu search to find an optimal solution, which is then considered to solve the original CAP problem. Our aim is to find out independent sets of cells by the multi-colouring method. Let us assume that *G* denotes the adjacency graph of the $n \times n$ matrix $C_{i,j}, i,j = 1, \dots, n$ and has a vertex set *V(G)* and edge set *E(G)*. We say that $(i,j) \in E(G)$ if and only if $c_{ij} \neq 0$. A node or vertex *i* is said to be connected to node *j* if $j \in adj(i)$ where $adj(i) = \{j | (i,j) \in E(G)\}$. The degree of vertex *i* is the size of its adjacency set $adj(i)$. A *proper colouring* of *G* is an assignment of colours to vertices such that no two end points of any edge share the same colour. A set S is said to be an independent set of *V(G)*

$$\text{if } i \in S \text{ then } (i,j) \in E(G) \text{ or } (j,i) \in E(G) \rightarrow j \notin S.$$

Thus, the elements in *S* cannot be connected among themselves. Independent sets can be obtained by applying the multi-colouring algorithm given in [22]. In this paper, we consider a simple greedy technique for obtaining a multi-colouring of an arbitrary graph. Initially a random permutation of the cells is obtained. The algorithm then assigns a colour of zero to each node *i*. Then, it traverses the graph in the natural order and assigns the smallest positive admissible colour to each node *i* visited. Here, an admissible colour is a colour not already assigned to any neighbour of node *i*. The end result of the algorithm is that each node *i* will be assigned the colour *Colour(i)*. The algorithm stops until all the nodes have been visited.

After having obtained the independent sets, we then form the reduced compatibility matrix. We then apply the tabu search algorithm to assign the frequencies to the reduced space.

Table 1. Compatibility Matrix for Problem 7

7	1	1	0	0	1	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0
1	7	1	1	0	0	1	1	1	1	0	0	0	0	1	1	1	0	0	0	0
1	1	7	1	1	0	0	1	1	1	1	0	0	0	0	1	1	1	0	0	0
0	1	1	7	1	0	0	0	1	1	1	1	0	0	0	0	1	1	0	0	0
0	0	1	1	7	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0
1	0	0	0	0	7	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0
1	1	0	0	0	1	7	1	1	0	0	0	1	1	1	1	0	0	1	0	0
1	1	1	0	0	1	1	7	1	1	0	0	0	1	1	1	1	0	1	1	0
1	1	1	1	0	0	1	1	7	1	1	0	0	0	1	1	1	1	1	1	1
0	1	1	1	1	0	0	1	1	7	1	1	0	0	0	1	1	1	0	1	1
0	0	1	1	1	0	0	0	1	1	7	1	0	0	0	0	1	1	0	0	1
0	0	0	1	1	0	0	0	0	1	1	7	0	0	0	0	0	1	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	7	1	1	0	0	0	0	0	0
1	0	0	0	0	1	1	1	0	0	0	0	1	7	1	1	0	0	1	0	0
1	1	0	0	0	1	1	1	1	0	0	0	1	1	7	1	1	0	1	1	0
1	1	1	0	0	0	1	1	1	1	0	0	0	1	1	7	1	1	1	1	1
0	1	1	1	0	0	0	1	1	1	1	0	0	0	1	1	7	1	1	1	1
0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	1	1	7	0	1	1
0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	7	1	1
0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	1	7	1
0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	1	7

Greedy Multi Colouring Algorithm

Set $Colour(i) = 0,\ i = 1, \dots, N$
for $i = 1 : N$
  $Colour(i) = \min\{l > 0 \mid l \neq Colour(j), \text{ for all } j \in adj(i)$
end

We shall illustrate our method by means of an example, problem 7 in Table 4 from the Philadelphia benchmark problems. The CAP, *P*, has been formulated on a 21-cell system whose compatibility matrix *C* and the demand vector D_1 is shown in Table 1 and Table 3 respectively. Denoting *S(i)* to be the set of all cells that are not connected, we illustrate an example for the problem described above. Suppose after applying the multi-colouring algorithm, we obtain the following: S (1) = {5; 7; 21} , S (2) = {10; 19}, S (3) = {9; 12; 14}, S (4) = {8; 13; 18}, S (5) = {3; 15}, S (6) = {6; 20}, S (7) = {1; 17}, S (8) = {2; 11} and S (9) = {4; 16}. From the set S we now construct the compatibility matrix $C' = c'(i', j')$ as follows. We get $c'(1,2) = 1$ being the maximum among all the $c_{i;j}$ where $i \in S(1)$ and $j \in S(2)$. We thus obtain the elements of C'as shown in Table 2. In this example, we can find that the demand for the elements in S(1) are 12, 30 and 25. The maximum demand is 30. Similarly the demand for S(2) = 40, S(3) = 45, S(4) = 30, S(5) 25, S(6) = 25, S(7)= 15, S(8) = 40 and S(9) = 15. Thus the modified demand vector D' = (30; 40; 45; 30; 25; 25; 15; 40; 15).

The new problem P' is represented by the following components:
1. a set S = {S(1),…, S (z)}; ($1 \leq i \leq z$) of z distinct nodes, where z is the number of $S(i)$ subsets.
2. a demand vector $D' = (d'_1, \dots, d'_z)$.
3. a compatibility matrix $C' = c'(i', j')$
4. a frequency assignment matrix F'.
5. a set of frequency separation constraints specified by the frequency separation matrix $|f'_{ik} - f'_{jl}|$ for all i, j, k, l (except for $i = j$ and $k = l$).

Once $P'$is constructed as above, the aim is to find an assignment F' for this P'. We describe how the CAP is solved using the tabu search algorithm. First, we define the blocked calls; they are the number of calls without an allocated frequency and the lower the number of blocked calls the better solution. Our aim is to minimize the number of blocked calls. We start our simulation by a random ordering or permutation of the cells and the tabu search is applied as shown in the algorithm below.

Procedure to apply the algorithm

Generate a random permutation order of the elements of S.
for j = 1 to *maxiter*
Apply intensification phase.
Apply *first(N).*
Apply diversification phase.
Calculate solution matrix F.
[calculate number of blocked calls].
If number of blocked calls = 0, break loop.
Calculate maxglob iterations > 5
end j
Calculate blocked calls.
Continue the process 200 times

Table 2. Compatibility Matrix for P' for Problem 7.

1	7	1	1	1	1	1	1	1
1	1	7	1	1	1	1	1	1
1	1	1	7	1	1	1	1	1
1	1	1	1	7	1	1	1	1
1	1	1	1	1	7	1	0	1
1	1	1	1	1	1	7	1	1
1	1	1	1	1	0	1	7	1
1	1	1	1	1	1	1	1	7
7	1	1	1	1	1	1	1	1

Now we need to apply the FAR algorithm to solve the original CAP problem with a view that our objective is to minimise the number of blocked calls. The assignment F' may or may not be admissible depending on the available bandwidth. To derive the required channels for the CAP, we have adapted the method found in [13].

They considered the following two cases:
1. An assignment F' is admissible: For this case, an admissible frequency for the original problem can be derived using $F'$and the following result [13]: Given the CAP problem P and the bandwidth, if the frequency assignments F' for P' are admissible, an admissible frequency can be derived from F'. To get an assignment F, all the cells in

$S(i)$; $1 \le i \le z$ are assigned the same set of channels. This assignment must satisfy the interference constraints because in P', $c'(i',j')$is the maximum among all the terms c_{ij}'s in C, where $i \in S(i)$ and $j \in S(j)$. This assignment must also satisfy the demand vector $D = d_i$, since we choose the maximum among all those cells found in *S(i)*. When applying this procedure, one also gets some redundant frequencies. Suppose if cell i has been assigned d'_i channels where the demand for that cell was d_i, and $d'_i > d(i)$, $r_i = (d' - d_i)_i$ number of frequencies remains unused or redundant in cell i.
2. Assignment F' is not admissible: For this case, the given bandwidth is not enough to satisfy the requirements for P'. Let us assume that F' satisfies the demand vector $D'' = (d''_i)$ instead of D', where $d''_i < d'_i$for some i. If we assign all the cells in *S(i)* the same set of channels to the cells in *S(i)*, there may be some blocked calls in some cells and redundant calls in some other cells. We denote the blocked calls and redundant frequencies as follows: BL = (b_i) and $R = (r_j)$, respectively, where $b_i = d_i - d''_i$ if $d''_i < d_i$ and 0 otherwise, and $r_j = (d''_j - d_j$, if $d''_j > d_j$ and 0 otherwise. We use these redundant frequencies in R and other available free channels to assign the blocked calls by using the FAR approach described in [16].

We give a brief description of the modified FAR algorithm. Let b_i be an unassigned requirement and Q be the set of already assigned frequencies. Suppose that from the set b_i there is no frequency available to be assigned without any conflict to the set Q. The modified FAR tries to assign a frequency in L (where L is the given list of available frequencies) to satisfy the requirement b_i with minimum change or perturbation on the present assignment Q. The main aim of modified FAR is to identify a minimal subset b_i's of Q, where each requirement can be reassigned simultaneously with an alternative feasible frequency, so that b_i can be assigned a frequency without conflict to the present assignment Q. We denote $B(b_i; f_i)$ to be the subset of requirements in Q, which are conflicting, and we assign frequency f_i to requirement b_i. In other words, f_i becomes a feasible frequency for b_i if the frequency assignments for $B(b_i; f_i)$ are undone. To identify one element of $S(b_i)$, we examine a sequence of f_i's such that each time a $B(b_i; f_i)$ is generated, we undo the corresponding portion of frequency assignment in Q and try to assign an alternative feasible frequency to each requirement of $B(b_i; f_i)$ by the unforced assignment operation. The unforced operation finds the lowest frequency in L, which is feasible to the present assignments in Q. If the frequency assignment of $B(b_i; f_i)$ is successfully made, $B(b_i; f_i)$ becomes $S(b_i)$ itself. In case such a frequency reassignment cannot be made for some $b_j \in B(b_i, f_i)$, one proceeds to identify $B(b_j ; f_j)$ and attempts to assign an alternative feasible frequency to each $b_k \in B(b_j ; f_j)$. Such $B(b_j ; f_j)$ are blockers at the second depth level. In our paper, we have used the (B1 - D1), (B2 - D1) and (B1 - D2).

This modified FAR actually assigns a free channel to an unassigned requirement, say $t \in BL$. We consider a channel to be free and suitable to be assigned to t even if it conflicts with the requirements of some other cells containing some redundant channels. However, when we choose such a channel for assigning it to t, we may need to undo some of the assignments in neighbouring cells and adjust the assignments in other cells as well as keeping the degree of perturbation as low as possible.

MULTIPLE OBJECTIVE APPROACH
The CAP has been formulated in such a way that we obtain a multi-objective dimension of the problem, which cannot be solved by using a single objective method. In other words, we minimise blocked calls and maximize redundant frequencies and these are objective functions that are conflicting. Thus, we shall use the NSGA II algorithm [8] to obtain the Pareto optimal solutions in one single run. The definitions and terms presented correspond to the mathematical formulations most widespread and adapted from [25]. A general multi-objective optimisation problem can be described as a vector function f that maps a tuple of m parameters (decision variables) to a tuple of n objectives, where x is called a decision vector and y denotes the objective vector. The set of solutions of a multi-objective optimization problem consists of all decision vectors, which cannot be improved, in any objective without degradation in other objectives. These vectors are known as Pareto optimal. Mathematically, the Pareto optimality can be explained as follows:

Assuming a maximisation problem and considering two decision vectors $a, b \in X$. We say that a dominates b, written as $a \succ b$ iff

$$\forall\, i \in \{1,2,\dots,n\}: f_i(a) \ge f_i(b) \wedge \exists\, j \in \{1,2,\dots,n\}: f_j(a) > f_j(b).$$

Furthermore, a covers b ($a \succcurlyeq b$) iff either $a \succ b$ or $a = b$. All decision vectors, which are not dominated by any other decision, are called nondominated or Pareto-optimal.
The family of all non-dominated alternate solutions is denoted as Pareto optimal set or Pareto optimal front. A mathematical definition of the Pareto optimality is as follows:

ISSN 2349-0292
Impact Factor 3.802

We say that a vector of decision variables $x^* \in \mathfrak{I}$ is Pareto optimal if there does not exist another $x \in \mathfrak{I}$ such that $f_j(x) > f_i(x_j^*)$ for all $i = 1, 2, \ldots, k$ and $f_j(x) > f_j(x^*)$ for at least one j. Here $\mathfrak{I}$ denotes the feasible region of the problem, in other words where the constraints are satisfied.

This definition means that x^* is Pareto optimal if there exists no feasible vector of decision variables $x \in \mathfrak{I}$, which would decrease some criterion without causing a simultaneous increase in at least one criterion. Unfortunately, this concept usually does not give not a single solution, but rather a set of solutions called the Pareto optimal set. The vectors x^* corresponding to the solutions included in the Pareto optimal set are called non-dominated. The plot of the objective functions, whose non-dominated vectors are in Pareto optimal set, is called the Pareto front. For a given multi-objective problem $f(x)$ and Pareto optimal set P^*, the Pareto front PF^* is defined as:

$$PF^* := \{u = f = (f_1(x), \ldots, f_k(x)) | x \in P^*\}.$$

EXPERIMENTAL RESULTS

The new method described earlier has been tested on eight benchmark problems. The cellular structure is shown in Figure 1. Different cases have been considered with different interference constraints and demand vectors as shown in Table 3 and Table 4. The parameters shown in the tables have been explained in section 2. The performance of our method described in the previous sections is tested as follows: for each problem, we choose three values M_1, M_2 and M_3, which is a maximum bandwidth. M_1 is a value less than the theoretical lower bound presented in table 4 [11], M_2 is equal to the lower bound and M_3 is greater than the theoretical lower bound. We first apply the tabu search algorithm and when we get the solution for the CAP, we count the number of blocked calls and redundant frequencies. For each value of $M_i, i = 1,2,3$, 200 iterations are performed. Using the 600 data set for each problem case, the NSGA II is applied to obtain the Pareto optimal solution. We thus obtain the set of figures (Figures 2-5). Since the aim of the paper was to show how computational time could be decreased, we repeat the same experiment with the reduction of the search space algorithm. Another set of Pareto optimal solutions are obtained and we give a table of comparison for the computational time.

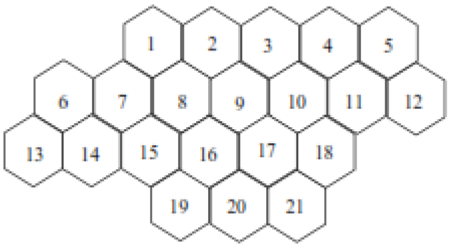


Figure 1: Structure of cellular systems

Table 3: Demand Vectors D_1 and D_2

	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}
D_1	8	25	8	8	8	15	18	52	77	28	13
D_2	5	5	5	8	12	25	30	25	30	40	40
	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	d_{20}	d_{21}	
D_1	15	31	15	36	57	28	8	10	13	8	
D_2	45	20	30	25	15	15	30	20	20	25	

The computation was done using a laptop with the following specifications: processor: Intel(R) Core(TM) 2 Duo CPU T5800 @ 2.00 Ghz 2.00 Ghz, RAM 2GB. To show that the reduction of the space has decreased the complexity and the computational time, we have run the different problems a number of times with the reduced space (SR) and without reducing the space (WSR) and the average time was calculated. From Table 5 it can be seen that there has been a consequent decrease in the computational time. In the plots obtained, the Pareto front is denoted by circles. The front denotes a tradeoff between the blocked calls and the redundant frequencies. These are helpful in case of sudden rise of demand and the optimal solution lie in the Pareto front for better decision making. It is observed that for problems which are easy to obtain the optimal solution in the Pareto front lie on the zero blocked calls axis. However, for the difficult cases that is for case 2 and case 6 the Pareto front is clear and distinct. In other words, we obtain better results for these harder cases with the multi-objective approach. In addition, it is an interesting observation that in cases of unpredicted rise in demands this new approach is indeed very suitable. If we compare the Pareto fronts obtained from the two methods, we find that for some cases, the Pareto points lie on the blocked calls axis and these are the cases where the computation time is low and furthermore there are redundant frequencies that can be assigned easily. For hard case problems, the manager can decide on the equilibrium on the number of redundant frequencies and block calls on the Pareto front that can be allowed in an unexpected change in the demand in certain cells.

Table 4. Different Problem Cases

Problem	1	2	3	4	5	6	7	8
a.c.c	1	2	1	2	1	2	1	2
c.s.c	5	5	7	7	5	5	7	7
D_1/D_2	D_1	D_1	D_1	D_1	D_2	D_2	D_2	D_2
Lower Bound	381	427	533	533	221	253	309	309

Table 5. Computational Time in seconds

Problem	1	2	3	4	5	6	7	8
WSR	3.1106E2	1.7795E3	2.9514E2	61.96353	3.8956E2	1.5733E3	1.011E2	53.36247
SR	68.42439	6.6045E2	43.3094	42.11038	53.015	58.015	79.17092	48.16189

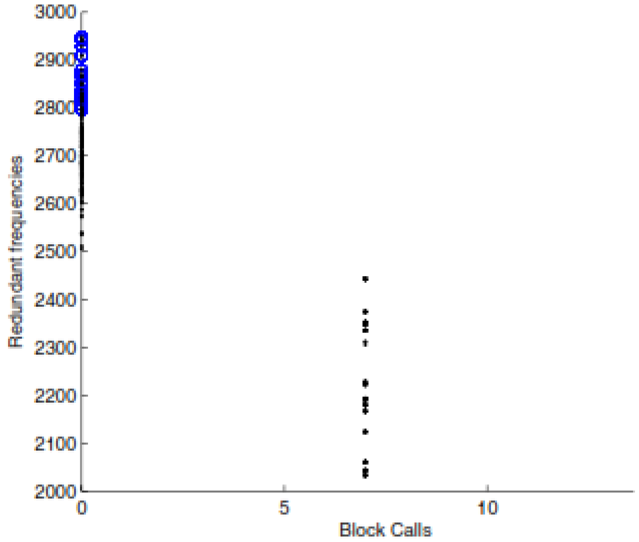
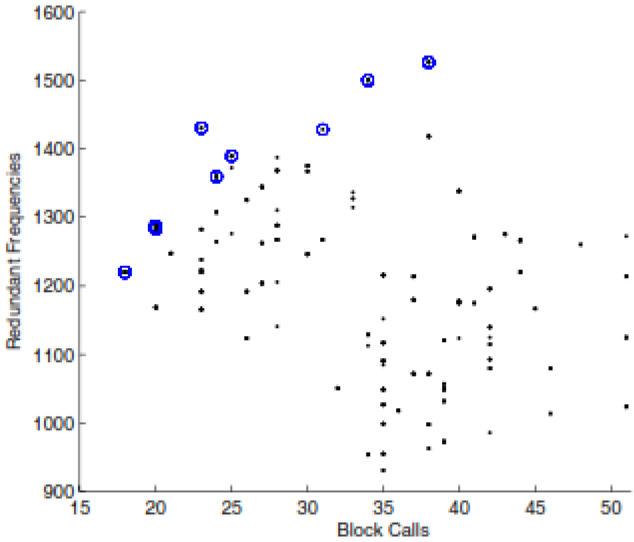


Figure 2: Pareto Front for Problem 1. (Table 4)



 ISSN 2349-0292
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Figure 3: Pareto Front for Problem 2. (Table 4)

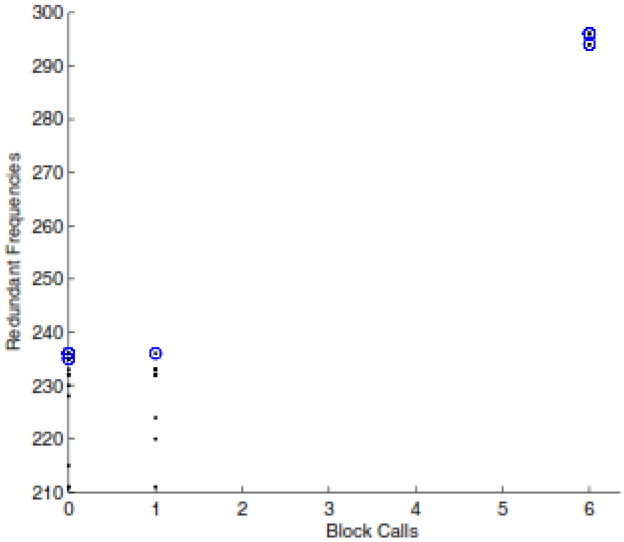

Figure 4: Pareto Front for Problem 1 using reduced space. (Table 4)

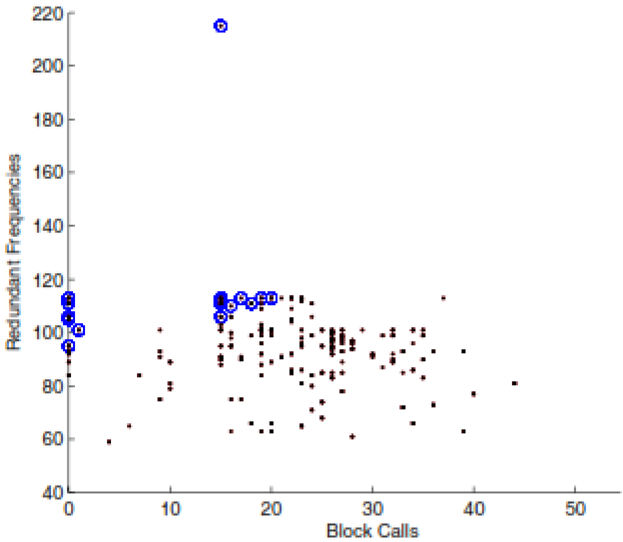

Figure 5: Pareto Front for Problem 2 using reduced space. (Table 4)

CONCLUSION

In this paper, we have considered a multi-objective approach to minimise the number of blocked calls and to maximise the redundant frequency. The results of the computations were presented in terms of Pareto fronts, which can be used by a system manager. The latter will decide for example in a certain event how many blocked calls can be allowed in the case there is a sudden rise in demand due to unexpected events. To gain in computational time in generating the Pareto fronts, a multi-colouring algorithm was developed to reduce the search space. Experimental results indicated that the proposed technique was computationally efficient and was particularly suitable for solving the harder problems in a much reduced time interval.

ISSN 2349-0292
Impact Factor 3.802

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