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CHARACTERIZATION OF INTUITIONISTIC FUZZY TOPOLOGICAL D - ALGEBRAS

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ABSTRACT

In this paper the concept of intuitionistic fuzzy topological d- algebras is studied, and some related properties are discussed.

KEYWORDS: d- algebras, fuzzy d- algebras, Intuitionistic fuzzy d- algebras, Intuitionistic fuzzy topological d- algebras.

AMS Subject Classification (2000): 06F35, 03F55, 03E72.03G25.

INTRODUCTION

The idea of intuitionistic fuzzy set was first published by K. T. Atanassov , as a generalization of the notion of fuzzy sets. In this paper, using the Atanassov's idea, we establish the notion of intuitionistic fuzzy *d*-algebras, equivalence relations on the family of all intuitionistic fuzzy *d*-algebras, and intuitionistic fuzzy topological *d*-algebras which are a generalization of the notion of fuzzy topological *d*-algebras, initiated by Jun and Kim . We investigate several properties, and show that the *d*-homomorphic image and preimage of an intuitionistic fuzzy topological *d*-algebra is an intuitionistic fuzzy topological *d*-algebra.

PRELIMINARIES

Definition 2.1 A *d-algebra* is a non-empty set X with a constant 0 and a binary operation "$*$" satisfying the following axioms:

(I) $x * x = 0$,
(II) $0 * x = 0$,
(III) $x * y = 0$ and $y * x = 0$ imply $x = y$

for all x, y, z in X .
A non-empty subset N of a d-algebra X is called a *d-subalgebra* of X if $x * y \in N$ for any $x, y \in N$. A mapping $\alpha : X \to Y$ of d-algebras is called a *d-homomorphism* if $\alpha(x * y) = \alpha(x) * \alpha(y)$ for all $x, y \in X$.

Definition 2.2 An *intuitionistic fuzzy set* (IFS) D in X is an object having the form

$$D = \{ x, \mu_D (x), \gamma_D (x) \quad | x \in X \}$$

where the functions $\mu_D : X \to [0, 1]$ and $\gamma_D : X \to [0, 1]$ denote the degree of membership (namely $\mu_D (x)$) and the degree of nonmembership (namely $\gamma_D (x)$) of each element $x \in X$ to the set D, respectively, and $0 \le \mu_D (x) + \gamma_D (x) \le 1$ for each $x \in X$.

For the sake of simplicity, we shall use the notation $D = (x, \mu_D , \gamma_D)$instead of $D = \{ x, \mu_D (x), \gamma_D (x) \mid x \in X \}$. Let f be a mapping from a set X to a set Y . If

$$B = \{ y, \mu_B (y), \gamma_B (y) \quad | y \in Y \}$$

is an IFS in Y , then the *preimage* of B under f , denoted by $f^{-1}(B)$, is the IFS in X defined by

$$f^{-1}(B) = \{ x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \quad | x \in X \},$$

and if $D = \{ x, \mu_D (x), \gamma_D (x) \mid x \in X \}$ is an IFS in X , then the *image* of D under f , denoted by $f (D)$, is the IFS in Y defined by

$$f (D) = \{ y, f_{sup}(\mu_D)(y), f_{inf} (\gamma_D)(y) \quad | y \in Y \},$$

where

$$f_{sup}(\mu_D)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \mu_D(x), & \text{if } f^{-1}(y) = \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$f_{inf}(\gamma_D)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \gamma_D(x), & \text{if } f^{-1}(y) = \emptyset, \\ 1, & \text{otherwise,} \end{cases}$$

for each $y \in Y$

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Definition 3.1. Let X be a d-algebra. An IFS, $D = (x, \mu_D, \gamma_D)$ in X is called an *intuitionistic fuzzy d-algebra* if it satisfies:

$$\mu_D(x * y) \geq \min\{\mu_D(x), \mu_D(y)\} \text{ and } \gamma_D(x * y) \leq \max\{\gamma_D(x), \gamma_D(y)\}$$

for all $x, y \in X$.

Proposition 3.2. *If an IFS,* $D = (x, \mu_D, \gamma_D)$ *in* X *is an intuitionistic fuzzy d-algebra of* X, *then* $\mu_D(0) \geq \mu_D(x)$ *and* $\gamma_D(0) \leq \gamma_D(x)$ *for all* $x \in X$.

Proof. Let $x \in X$. Then $\mu_D(0) = \mu_D(x * x) \geq \min\{\mu_D(x), \mu_D(x)\} = \mu_D(x)$ and $\gamma_D(0) = \gamma_D(x * x) \leq \max\{\gamma_D(x), \gamma_D(x)\} = \gamma_D(x)$.

Theorem 3.3. *If* $\{D_i \mid i \in \Lambda\}$ *is an arbitrary family of intuitionistic fuzzy d-algebras of* X, *then* $\cap D_i$ *is an intuitionistic fuzzy d-algebra of* X,

where $\cap D_i = \{ x, \wedge\mu_{Di}(x), \vee\gamma_{Di}(x) \mid x \in X \}$.

Proof. Let $x, y \in X$. Then

$$\wedge\mu_{Di}(x * y) \geq \wedge(\min\{\mu_{Di}(x), \mu_{Di}(y)\}) = \min\{ \mu_{Di}(x), \wedge\mu_{Di}(y)\}$$

and

$$\vee\gamma_{Di}(x * y) \leq \vee(\max\{\gamma_{Di}(x), \gamma_{Di}(y)\}) = \max\{ \gamma_{Di}(x), \vee\gamma_{Di}(y)\}.$$ Hence $\cap D_i = x, \wedge\mu_{Di}$, $\vee\gamma_{Di}$ is an intuitionistic fuzzy d-algebra of X. □

Theorem 3.4. *If an IFS* $D = (x, \mu_D, \gamma_D)$ *in* X *is an intuitionistic fuzzy d-algebra of* X, *then so is D, where* $D = \{ x, \mu_D(x), 1 - \mu_D(x) \mid x \in X \}$.

Proof. It is sufficient to show that $\bar{\mu}_D$ satisfies the second condition in Definition 3.1.

Let $x, y \in X$. Then

$$\begin{aligned} \bar{\mu}_D(x * y) = 1 - \mu_D(x * y) &\leq 1 - \min\{\mu_D(x), \mu_D(y)\} \\ &= \max\{1 - \mu_D(x), 1 - \mu_D(y)\} \\ &= \max\{\bar{\mu}_D(x), \bar{\mu}_D(y)\}. \end{aligned}$$

Hence D is an intuitionistic fuzzy d-algebra of X. □

Theorem 3.5. *If an IFS* $D = (x, \mu_D, \gamma_D)$ *in* X *is an intuitionistic fuzzy d-algebra of* X, *then the sets* $X_\mu := \{x \in X \mid \mu_D(x) = \mu_D(0)\}$ and $X_\gamma := \{x \in X \mid \gamma_D(x) = \gamma_D(0)\}$ *are d-subalgebras of* X.

Proof. Let $x, y \in X_\mu$. Then $\mu_D(x) = \mu_D(0) = \mu_D(y)$, and so

$$\mu_D(x * y) \geq \min\{\mu_D(x), \mu_D(y)\} = \mu_D(0)$$

By using Proposition 3.2, we know that $\mu_D(x * y) = \mu_D(0)$ or equivalently $x * y \in X_\mu$. Now let $x, y \in X_\gamma$. Then

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$$\gamma_D(x * y) \leq \max\{\gamma_D(x), \gamma_D(y)\} = \gamma_D(0)$$

and by applying Proposition 3.2 we conclude that
$\gamma_D(x * y) = \gamma_D(0)$ and hence $x * y \in X_\gamma$. □

Definition 3.6. Let $D = (x, \mu_D, \gamma_D)$ be an IFS in X and let $t \in [0, 1]$. Then the set
$U(\mu_D, t) := \{x \in X \mid \mu_D(x) \geq t\}$ (resp. $L(\gamma_D, t) := \{x \in X \mid \gamma_D(x) \leq t\}$) is called a *μ-level t-cut* (resp. *γ-level t-cut*) of D.

Theorem 3.7. *If an IFS $D = (x, \mu_D, \gamma_D)$ in X is an intuitionistic fuzzy d-algebra of X, then the μ-level t-cut and γ-level t-cut of D are d-subalgebras of X for every $t \in [0, 1]$ such that $t \in \mathrm{Im}(\mu_D) \cap \mathrm{Im}(\gamma_D)$,* which are called a *μ-level d-subalgebra* and a *γ-level d-subalgebra* respectively.

Proof. Let $x, y \in U(\mu_D, t)$. Then $\mu_D(x) \geq t$ and $\mu_D(y) \geq t$. It follows that $\mu_D(x * y) \geq \min\{\mu_D(x), \mu_D(y)\} \geq t$ so that $x * y \in U(\mu_D, t)$. Hence $U(\mu_D, t)$ is a d-subalgebra of X . Now let $x, y \in L(\gamma_D, t)$. Then $\gamma_D(x * y) \leq \max\{\gamma_D(x), \gamma_D(y)\} \leq t$ and so $x * y \in L(\gamma_D, t)$. Therefore $L(\gamma_D, t)$ is a d-subalgebra of X .

Theorem 3.8 *Let $D = (x, \mu_D, \gamma_D)$ be an* IFS *in X such that the sets $U(\mu_D, t)$ and $L(\gamma_D, t)$ are d-subalgebras of X . Then $D = (x, \mu_D, \gamma_D)$ is an intuitionistic fuzzy d-algebra of X .*

Proof. Assume that there exist $x_0, y_0 \in X$ such that $\mu_D(x_0 * y_0) < \min\{\mu_D(x_0), \mu_D(y_0)\}$.
Let

$$\overline{\mu}\ t_0 := 2\ \mu_D(x_0 * y_0) + \min\{\mu_D(x_0), \mu_D(y_0)\} .$$

Then $\mu_D(x_0 * y_0) < t_0 < \min\{\mu_D(x_0), \mu_D(y_0)\}$ and so $x_0 * y_0 \notin U(\mu_D, t_0)$, but $x_0, y_0 \in U(\mu_D, t_0)$. This is a contradiction, and therefore

$$\mu_D(x * y) \geq \min\{\mu_D(x), \mu_D(y)\} \text{ for all } x, y \in X .$$

Now suppose that

$$\gamma_D(x_0 * y_0) > \max\{\gamma_D(x_0), \gamma_D(y_0)\} \text{ for some } x_0, y_0 \in X$$

. Taking

$$\overline{\gamma}\ s_0 := 2\ \gamma_D(x_0 * y_0) + \max\{\gamma_D(x_0), \gamma_D(y_0)\} ,$$

then $\max\{\gamma_D(x_0), \gamma_D(y_0)\} < s_0 < \gamma_D(x_0 * y_0)$. It follows that $x_0, y_0 \in L(\gamma_D, s_0)$ and $x_0 * y_0 \notin L(\gamma_D, s_0)$, a contradiction. Hence

$$\gamma_D(x * y) \leq \max\{\gamma_D(x), \gamma_D(y)\}$$

for all $x, y \in X$. This completes the proof. □

Theorem 3.9. *Any d-subalgebra of X can be realized as both a μ-level d-subalgebra and a γ-level d-subalgebra of some intuitionistic fuzzy d-algebra of X.*

Proof. Let S be a d-subalgebra of X and let μ_D and γ_D be fuzzy sets in X defined by

$$\mu_D(x) := \begin{cases} t, & \text{if } x \in S, \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\gamma_D(x) := \begin{cases} s, & \text{if } x \in S, \\ 1, & \text{otherwise,} \end{cases}$$

for all $x \in X$ where t and s are fixed numbers in (0, 1) such that $t + s < 1$. Let $x, y \in X$. If $x, y \in S$, then $x * y \in S$. Hence $\mu_D(x * y) = \min\{\mu_D(x), \mu_D(y)\}$ and $\gamma_D(x * y) = \max\{\gamma_D(x), \gamma_D(y)\}$. If at least one of x and y does not belong to S, then at least one of

$\mu_D(x)$ and $\mu_D(y)$ is equal to 0, and at least one of $\gamma_D(x)$ and $\gamma_D(y)$ is equal to 1. It follows that

$$\mu_D(x * y) \geq 0 = \min\{\mu_D(x), \mu_D(y)\},$$

$$\gamma_D(x * y) \leq 1 = \max\{\gamma_D(x), \gamma_D(y)\}.$$

Hence $D = (x, \mu_D, \gamma_D)$ is an intuitionistic fuzzy d-algebra of X. Obviously, $U(\mu_D, t) = S = L(\gamma_D, s)$. This completes the proof.

Theorem 3.10. *Let α be a d-homomorphism of a d-algebra X into a d-algebra Y and B an intuitionistic fuzzy d-algebra of Y. Then $\alpha^{-1}(B)$ is an intuitionistic fuzzy d-algebra of X.*

Proof. For any $x, y \in X$, we have

$$\mu_{\alpha^{-1}(B)}(x * y) = \mu_B(\alpha(x * y)) = \mu_B(\alpha(x) * \alpha(y)) \geq \min\{\mu_B(\alpha(x)), \mu_B(\alpha(y))\}$$

1) $\min\{\mu_{\alpha^{-1}(B)}(x), \mu_{\alpha^{-1}(B)}(y)\}$

and

$$\gamma_{\alpha^{-1}(B)}(x * y) = \gamma_B(\alpha(x * y)) = \gamma_B(\alpha(x) * \alpha(y))$$

\- $\max\{\gamma_B(\alpha(x)), \gamma_B(\alpha(y))\}$

1. $\max\{\gamma_{\alpha^{-1}(B)}(x), \gamma_{\alpha^{-1}(B)}(y)\}$.

Hence $\alpha^{-1}(B)$ is an intuitionistic fuzzy d-algebra in X. □

Theorem 3.11. *Let α be a d-homomorphism of a d-algebra X onto a d-algebra Y. If $D = (x, \mu_D, \gamma_D)$ is an intuitionistic fuzzy d-algebra of X, then $\alpha(D) = y, \alpha_{sup}(\mu_D), \alpha_{inf}(\gamma_D)$ is an intuitionistic fuzzy d-algebra of Y.*

Proof. Let $D = (x, \mu_D, \gamma_D)$ be an intuitionistic fuzzy topological d-algebra in X and let $y_1, y_2 \in Y$. Noticing that

$$\{x_1 * x_2 \mid x_1 \in \alpha^{-1}(y_1) \text{ and } x_2 \in \alpha^{-1}(y_2)\} \subseteq \{x \in X \mid x \in \alpha^{-1}(y_1 * y_2)\},$$

we have

$\alpha_{sup}(\mu_D)(y_1 * y_2)$

1. $\sup\{\mu_D(x) \mid x \in \alpha^{-1}(y_1 * y_2)\}$

$\geq \sup\{\mu_D(x_1 * x_2) \mid x_1 \in \alpha^{-1}(y_1) \text{ and } x_2 \in \alpha^{-1}(y_2)\}$

$\geq \sup \min\{\mu_D(x_1), \mu_D(x_2)\} \mid x_1 \in \alpha^{-1}(y_1) \text{ and } x_2 \in \alpha^{-1}(y_2)$

$= \min \sup\{\mu_D(x_1) \mid x_1 \in \alpha^{-1}(y_1)\}, \sup\{\mu_D(x_2) \mid x_2 \in \alpha^{-1}(y_2)\}$

$= \min\{\alpha_{sup}(\mu_D)(y_1), \alpha_{sup}(\mu_D)(y_2)\}$

and

$\alpha_{inf}(\gamma_D)(y_1 * y_2)$

$= \inf\{\gamma_D(x) \mid x \in \alpha^{-1}(y_1 * y_2)\}$

$\leq \inf\{\gamma_D(x_1 * x_2) \mid x_1 \in \alpha^{-1}(y_1) \text{ and } x_2 \in \alpha^{-1}(y_2)\}$

$\leq \inf \max\{\gamma_D(x_1), \gamma_D(x_2)\} \mid x_1 \in \alpha^{-1}(y_1) \text{ and } x_2 \in \alpha^{-1}(y_2)\}$

$= \max \inf\{\gamma_D(x_1) \mid x_1 \in \alpha^{-1}(y_1)\}, \inf\{\gamma_D(x_2) \mid x_2 \in \alpha^{-1}(y_2)\}$

$= \max\{\alpha_{inf}(\gamma_D)(y_1), \alpha_{inf}(\gamma_D)(y_2)\}$.

=

Hence $\alpha(D) = y, \alpha_{sup}(\mu_D), \alpha_{inf}(\gamma_D)$ is an intuitionistic fuzzy d-algebra in Y.

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Coker[3] generalized the concept of fuzzy topological space, first initiated by Chang [2]to the case of intuitionistic fuzzy sets as follows.

Definition 4.1 An *intuitionistic fuzzy topology* (IFT for short) on a nonempty set X is a family Φ of IFSs in X satisfying the following axioms:

(T1) $0_\sim, 1_\sim \in \Phi$,
(T2) $G_1 \cap G_2 \in \Phi$ for any $G_1, G_2 \in \Phi$,
(T3) $G_i \in \Phi$ for any family $\{G_i : i \in J\} \subseteq \Phi$. $i \in J$

In this case the pair (X, Φ) is called an *intuitionistic fuzzy topological space* (IFTS for short) and any IFS in Φ is called an *intuitionistic fuzzy open set* (IFOS for short) in X .

Definition 4.2 Let (X, Φ) and (Y, Ψ) be two IFTSs. A mapping $f : X \to Y$ is said to be *intuitionistic fuzzy coninuous* if the preimage of each IFS in Ψ is an IFS in Φ; and f is said to be *intuitionistic fuzzy open* if the image of each IFS in Φ is an IFS in Ψ.

Definition 4.3. Let D be an IFS in an IFTS (X, Φ). Then the *induced intuitionistic fuzzy topology* (IIFT for short) on D is the family of IFSs in D which are the intersection with D of IFOSs in X . The IIFT is denoted by Φ_D , and the pair (D, Φ_D) is called an *intuitionistic fuzzy subspace* of (X, Φ).

Definition 4.4. Let (D, Φ_D) and (B, Ψ_B) be intuitionistic fuzzy subspaces of IFTSs (X, Φ) and (Y, Ψ), respectively, and let $f : X \to Y$ be a mapping. Then f is a mapping of$D$ into B if $f(D) \subset B$. Furthermore, f is said to be *relatively intuitionistic fuzzy continuous* if for each IFS V_B in Ψ_B , the intersection $f^{-1}(V_B) \cap D$ is an IFS in Φ_D ; and f is said to be *relatively intuitionistic fuzzy open* if for each IFS U_D in Φ_D , the image $f(U_D)$ is an IFS in Ψ_B .

Proposition 4.5. *Let (D, Φ_D) and (B, Ψ_B) be intuitionistic fuzzy subspaces of IFTSs (X, Φ) and (Y, Ψ) respectively, and let f be an intuitionistic fuzzy continuous mapping of X into Y such that $f(D) \subset B$. Then f is relatively intuitionistic fuzzy continuous mapping of D into B.*

Proof. Let V_B be an IFS in Ψ_B . Then there exists $V \in \Psi$ such that $V_B = V \cap B$. Since f is intuitionistic fuzzy continuous, it follows that $f^{-1}(V)$ is an IFS in Φ. Hence

$$f^{-1}(V_B) \cap D = f^{-1}(V \cap B) \cap D = f^{-1}(V) \cap f^{-1}(B) \cap D = f^{-1}(V) \cap D$$

is an IFS in Φ_D . This completes the proof. □

For any d-algebra X and any element $a \in X$ we use a_r to denote the self map of X defined by $a_r(x) = x * a$ for all $x \in X$.

Definition 4.6. Let X be a d-algebra, Φ an IFT on X and D an intuitionistic fuzzy d-algebra with IIFT Φ_D . Then D is called an *intuitionistic fuzzy topological d-algebra* if for each $a \in X$ the mapping $a_r : (D, \Phi_D) \to (D, \Phi_D)$, $x \to x * a$, is relatively intuitionistic fuzzy continuous.

Theorem 4.7. *Given d-algebras X and Y, and a d-homomorphism $\alpha : X \to Y$, let Φ and Ψ be the IFTs on X and Y respectively such that $\Phi = \alpha^{-1}(\Psi)$. If B is an intuitionistic fuzzy topological d-algebra in Y , then $\alpha^{-1}(B)$ is an intuitionistic fuzzy topological d-algebra in X .*

Proof. Let $a \in X$ and let U be an IFS in $\Phi_{\alpha^{-1}(B)}$. Since α is an intuitionistic fuzzy con-tinuous mapping of (X, Φ) into (Y, Ψ), it follows from Proposition 4.5 that α is a relatively intuitionistic fuzzy continuous mapping of $(\alpha^{-1}(B), \Phi_{\alpha^{-1}(B)})$ into (B, Ψ_B). Note that there exists an IFS V in Ψ_B such that $\alpha^{-1}(V) = U$. Then

$$\mu_{a_r^{-1}(U)}(x) = \mu_U(a_r(x)) = \mu_U(x * a) = \mu_{\alpha^{-1}(V)}(x * a)$$
$$= \mu_V(\alpha(x * a)) = \mu_V(\alpha(x) * \alpha(a))$$

and

$$\gamma_{a_r^{-1}(U)}(x = \gamma_U(a_r(x)) = \gamma_U(x * a) = \gamma_{\alpha^{-1}(V)}(x * a)$$
$$= \gamma_V(\alpha(x * a)) = \gamma_V(\alpha(x) * \alpha(a)).$$

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Since B is an intuitionistic fuzzy topological d-algebra in Y, the mapping

$$b_r : (B, \Psi_B) \to (B, \Psi_B), \; y \to y * b$$

is relatively intuitionistic fuzzy continuous for every $b \in Y$. Hence

$$\mu_{a^-{}_r{}^{1}(U)}(x) = \mu_V(\alpha(x) * \alpha(a)) = \mu_V(\alpha(a)_r(\alpha(x)))$$

$$= \mu_{\alpha(a)^-{}_r{}^{1}(V)}(\alpha(x)) = \mu_{\alpha^{-1}(\alpha(a)^-{}_r{}^{1}(V))}(x)$$

and

$$\gamma_{a^-{}_r{}^{1}(U)}(x) = \gamma_V(\alpha(x) * \alpha(a)) = \gamma_V(\alpha(a)_r(\alpha(x)))$$

$$= \gamma_{\alpha(a)^-{}_r{}^{1}(V)}(\alpha(x)) = \gamma_{\alpha^{-1}(\alpha(a)^-{}_r{}^{1}(V))}(x).$$

Therefore $a^-{}_r{}^{1}(U) = \alpha^{-1}(\alpha(a)^-{}_r{}^{1}(V))$, and so

$$a^-{}_r{}^{1}(U) \cap \alpha^{-1}(B) = \alpha^{-1}(\alpha(a)^-{}_r{}^{1}(V)) \cap \alpha^{-1}(B)$$

is an IFS in $\Phi_{\alpha^{-1}(B)}$. This completes the proof.

Theorem 4.8. *Given d-algebras X and Y, and a d-isomorphism α of X to Y, let Φ and Ψ be the IFTs on X and Y respectively such that α(Φ) = Ψ. If D is an intuitionistic fuzzy topological d-algebra in X, then α(D) is an intuitionistic fuzzy topological d-algebra in Y.*

Proof. It is sufficient to show that the mapping

$$b_r : (\alpha(D), \Psi_{\alpha(D)}) \to (\alpha(D), \Psi_{\alpha(D)}), \; y \to y * b$$

is relatively intuitionistic fuzzy continuous for each $b \in Y$. Let U_D be an IFS in Φ_D. Then there exists an IFS U in Φ such that $U_D = U \cap D$. Since α is one-one, it follows that

$$\alpha(U_D) = \alpha(U \cap D) = \alpha(U) \cap \alpha(D)$$

which is an IFS in $\Psi_{\alpha(D)}$. $V_{\alpha(D)}$ be an IFS in $\Psi_{\alpha(D)}$.
This shows that α is relatively intuitionistic fuzzy open. Let The surjectivity of α implies that for each $b \in Y$ there exists

$a \in X$ such that $b = \alpha(a)$. Hence

$$\mu_{\alpha^{-1}(b^-{}_r{}^{1}(V_{\alpha(D)}))}(x) = \mu_{\alpha^{-1}(\alpha(a)^-{}_r{}^{1}(V_{\alpha(D)}))}(x) = \mu_{\alpha(a)^-{}_r{}^{1}(V_{\alpha(D)})}(\alpha(x))$$
$$= \mu_{V_{\alpha(D)}}(\alpha(a)_r(\alpha(x))) = \mu_{V_{\alpha(D)}}(\alpha(x) * \alpha(a))$$
$$= \mu_{V_{\alpha(D)}}(\alpha(x * a)) = \mu_{\alpha^{-1}(V_{\alpha(D)})}(x * a)$$

$$= \mu_{\alpha^{-1}(V_{\alpha(D)})}(a_r(x)) = \mu_{a^-{}_r{}^{1}(\alpha^{-1}(V_{\alpha(D)}))}(x)$$

and

$$\gamma_{\alpha^{-1}(b^-{}_r{}^{1}(V_{\alpha(D)}))}(x) = \gamma_{\alpha^{-1}(\alpha(a)^-{}_r{}^{1}(V_{\alpha(D)}))}(x) = \gamma_{\alpha(a)^-{}_r{}^{1}(V_{\alpha(D)})}(\alpha(x))$$
$$= \gamma_{V_{\alpha(D)}}(\alpha(a)_r(\alpha(x))) = \gamma_{V_{\alpha(D)}}(\alpha(x) * \alpha(a))$$

$$= \gamma_{V_{\alpha(D)}}(\alpha(x * a)) = \gamma_{\alpha^{-1}(V_{\alpha(D)})}(x * a)$$

$$= \gamma_{\alpha^{-1}(V_{\alpha(D)})}(a_r(x)) = \gamma_{a^-{}_r{}^{1}(\alpha^{-1}(V_{\alpha(D)}))}(x).$$

Therefore $\alpha^{-1}(b^-{}_r{}^{1}(V_{\alpha(D)})) = a^-{}_r{}^{1}(\alpha^{-1}(V_{\alpha(D)}))$. By hypothesis, the mapping

$$a_r : (D, \Phi_D) \to (D, \Phi_D), \; x \to x * a$$

is relatively intuitionistic fuzzy continuous and α is a relatively intuitionistic fuzzy continuous map: $(D, \Phi_D) \to (\alpha(D), \Psi_{\alpha(D)})$. Thus

$$\alpha^{-1}(b^-{}_r{}^{1}(V_{\alpha(D)})) \cap D = a^-{}_r{}^{1}(\alpha^{-1}(V_{\alpha(D)})) \cap D \text{ is an IFS in } \Phi_D. \text{ Since}$$

α is relatively intuitionistic fuzzy open,

$\alpha(\alpha^{-1}(b^{-}{}_{r}{}^{1}(V_{\alpha(D)})) \cap D) = b^{-}{}_{r}{}^{1}(V_{\alpha(D)}) \cap \alpha(D)$ is an IFS in $\Psi_{\alpha(D)}$.

This completes the proof.

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